SOME ASPECTS OF THERMAL ANALYSIS OF AN INTERIM RADIOACTIVE WASTE STORAGE FACILITY

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CERTIFICATE

It is to certify that the work contained in the thesis entiteled " SOME ASPECTS OF THERMAL ANALYSIS OF AN INTERIM RADIOACTIVE WASTE STORAGE FACILITY " has been carried under our supervision and that this work has not been submitted elsewhere for the award of a degree .

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ABSTRACT

The solidified waste product produced from a fuel reprocessing plant has high heat generation rates . It is therefore necessary to store these wastes for an interim period before final disposal . This is stored in well sealed containers to prevent the leakage of activity . These containers are continuously cooled to dissipate the decay heat .

For this purpose natural or forced circulation of air is used . For a given set of conditions heat generation rate, heat transfer coefficient etc. have been estimated . The shape of storage canisters is cylindrical . These calculations have been done for different orientations of canisters ,e.g. ,vertical, horizontal , vertical with sleeve (annular) and cross flow across a tube bundle .

After this the temperature distributions radially as well as along the height are evaluated using a finite difference scheme requiring a comprehensive computer programme .

Finally accident analysis has been done i.e. if the entire system fails and air flow is completely choked . Then how long would it take for the temperatures to rise above critical limits , within which maintenance personnel should be able to repair the system .

ACKNOWLEDGEMENTS

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NOTATIONS

HAW	High Active Wastes
MAW	Medium Active Wastes
LAW	Low Active Wastes
TRU	Transuranic Wastes
LMAW	Low and Medium Active Wastes
LWR	Liquid Water Reactor
T_{∞}	Ambient Temperature
T_w	Wall Temperature
T_f	Film Temperature
A	Area
d	Diameter
l	Length
N	Number of Storage Units
Q	Heat Load
C_p	Specific Heat
C	Heat Capacity
$\dot{m}$	Mass Flow Rate
ρ_{∞}	Density of Air
P	Pressure
R	Gas Constant
V	Volume Flow Rate
v	Flow Velocity
ΔP	Pressure Difference
f	Friction factor
G_{max}	Mass Velocity at the Minimum Area
ν	Kinematic Viscosity
μ	Dynamic Viscosity

Re	Reynolds Number
Pr	Prandtl Number
Nu	Nusselt Number
Gr	Grashof Number
S_n	Pitch of Storage Units
K_f	Thermal conductivity of air at Film Temperature
U_∞	Free Stream Velocity
h	Convection Heat Transfer coefficient
q'''	Heat generation rate per unit volume
x	distance
δ	Annular gap
r	Radius
z	Height
τ	Time
Ø	Base angle
α	Thermal Diffusivity
Δr	Radial increment
Δz	Height wise increment

Chapter 1

INTRODUCTION AND WASTE STORAGE FACILITY

1.1 INTRODUCTION

The spent fuel derived from a nuclear reactor is reprocessed to extract the unused fuel from it . The wastes coming after reprocessing are characterized by high heat generation rates and are highly radioactive , especially if it is of high active type . Therefore it becomes necessary to store these wastes for an interim period of time . During which a major portion of decay heat is dissipated so that an inert and stable form is available for the final disposal . This is known as conditioning of the wastes , after which it becomes easier to handle , transport and store them safely . After the interim storage they are disposed to the ultimate disposal sites with lesser costs .

The storage facility where these wastes are stored for an interim period has to be designed very carefully . All the thermal , biological and structural conditions should be satisfied by it . Thermally it should be able to provide continuous cooling of the product in order to avoid melting , which may lead to an extremely dangerous situation . Biologically it should be able to contain radiations coming out of wastes , and structurally

it should be able to withstand tremors , etc. . To ensure the integrity of wastes and its shielding canister , constant surveillance is required . It is expected that thermal and radiation conditions are stabilized after a period of about 10 years , but to be on safe side an additional period of about 10 years is provided . Hence the storage facility should have sufficient space to store the wastes generated over a period of 20 years . [1,2]

1.2 RADIOACTIVE WASTES [3,10]

Basically there are following four kinds of radioactive wastes -

1. High active wastes (HAW)
2. Medium active wastes (MAW)
3. Low active wastes (LAW)
4. Transuranic wastes (TRU)

HIGH ACTIVE WASTES

HAW is a term used for wastes derived from partitioning fission products and transuranics from uranium and plutonium during the reprocessing of used nuclear fuel. It may be in the form of sludge , calcine , or other products generated in treating liquid HAW . They require cooling and shielding both and great care is required while storing them .The radioactivity level of these wastes is 100 $\mu ci\ ml^{-1}$ and above.

MEDIUM ACTIVITY WASTES

MAW is the term used for those intermediate level radioactive wastes which do not require cooling but require shielding and can be safely stored but for which disposal facilities at present are not available, and which do not fall within the category of heat generating wastes HAW . The radioactivity level of these wastes is 10^{-5} to 100 μci ml^{-1}.

LOW ACTIVITY WASTES

LAW are defined as radioactive wastes which do not require shielding and cooling and are usually classified in terms of the concentration of radioactivity they contain . Although LAW are those materials with a low level of activity which do not often require treatment before disposal , they may contain some low and potentially hazardous cocentrations of radionuclides . Their radioactivity level is less than 10^{-5} μci ml^{-1} .

TRANSURANIC WASTES

Transuranic wastes are those elements containing isotopes beyond uranium in the periodic table . these are produced almost entirely by the irradiation of U & Th in nuclear reactors . Characteristically these are α - emitters whose decay schemes follow the natural U and Th decay

schemes . Examples of these isotopes are -
plutonium^{239} , Armecium^{243} , Curium^{244,245} , Neptunium^{237} (very long half life , 2.14 million years) .

1.3 WASTES DURING REPROCESSING [3]

Fissioning of nuclear fuel in a reactor creates highly radioactive materials which are almost all retained in the used fuel elements when removed from the reactor . When the used fuel is reprocessed to recover unburnt uranium and plutonium , most of the radioctivity is retained in the liquid wastes which are later converted into solids .

Reprocessing of used fuel from a nuclear power facility of 1 GWe operating at a plant factor of 70 percent would generate 5000 to 7000 kg of solidified HAW .

About 5000 liters of primary HAW solutions are used in the reprocessing of 1000 kg of used uranium fuel. The volume would be reduced to 1.1 liters by evaporation before storage in the stainless - steel storage tanks . Prior to the actual solidification process , the volume would be further reduced by evaporation to some 0.38 liters .

Table 1.1 shows a typical inventory of radioactive wastes arising from reprocessing -

TABLE 1.1

Inventory of radioactive wastes during reprocessing

Reprocessing wastes	Waste material generated (m^3/Gg HM)	Plutonium content (Kg/Gg HM)
HAW	500	50
LMAW	1500	15
Combustible	1845	8
Non - Combustible	1500	15
Cladding spacers	350	15
Dissolver residues	50	0

In most of the fuel reprocessing plants , gaseous effluents generated are particularly important , even though some isotope decay quickly . In particular iodine -129 which has only a 0.8 percent fission yield has a half life of 17 Myr , carbon -14 - 5500 years , krypton - 3950 days and tritium - 12 years . BWR fuel elements contain about one percent unburnt uranium -235 , 0.5 percent plutonium isotopes , mainly plutonium 239 , and the balance about 95 percent nonfissile uranium -238 .

1.4 ALTERNATIVES FOR WASTE STORAGE [4,5]

Several designs have been considered for interim storage . Use of water ponds for providing interim storage for the solidified waste is an alternative . One option is to use air cooling on the basis of detailed techno - economic consideration , both air and water are acceptable .Water of course is a better cooling medium because of its high heat capacity , as compared to that of air . However , consideration of corrosion , need for demineralised & chemically adjusted water & high make up water are some of the disadvantages . The other alternative uses natural convective air cooling for air cooled storage the waste canister is enclosed in a secondary container to ensure maximum safety against accidental release of activity in the event of canister failures . Cross flow of air with stack induced natural circulation is preferred . However a back up system for forced supply might be required to cater for unforeseen eventualities leading to breakdown of natural air circulation .

1.5 DESIGN OF SOLIDIFIED WASTE STORAGE FACILITY [1]

The solidified radioactive wastes produced from reprocessing plant are stored in fully sealed , stainless steel canister . The air cooled storage utilizes decay heat & a suitably designed stack to provide the driving force required for the movement of air through the storage

vault . In effect the waste heat increases the air temperature causing an upward movement of air due to bouyant force .

The design depends upon the rate at which containers containing solidified highly radioactive wastes are received from the waste immobilization plant . In fact the waste from the reprocessing plant is cooled for 900 days before solidification . The liquid waste contains about 675 ci $lit.^{-1}$ of β , γ activity generating about 2 watts $lit.^{-1}$ of decay heat . The specific activity of the vitrified product is around 13,500 ci $lit.^{-1}$ generating about 40 watts $lit.^{-1}$. It is important to note that actinides have not been taken in to consideration in these estimates since they are relatively less important within the time frame of interim storage . The actinides assume primary importance for long term storage and disposal condition , for instance in the Tarapur Storage Facility , which has been designed to store wastes generated over a period of 20 years , actinides have not been taken in to account .

To meet the design objectives a secondary containment called storage unit is provided to the storage canister . For canister simple cylindrical geometry is selected . Finned or annular geometry is not envisaged in view of low heat generation rate of the liquid waste due to moderate burn up of the spent fuel . For enclosing the canisters in the storage unit three arrangements are considered :

-Axial stacking

Radial

-Combination of above two

Second and third will reduce considerably overall storage positions. But it has got disadvantages of poor heat transfer efficiency leading to uneven temperature distribution in the canisters. From heat transfer point of view axial stacking is same as single storage unit with increased height. In the solid storage surveillance facility at Tarapur two canisters are stacked in a single storage unit. The storage unit is completely sealed by welding the lid on.

Thus in a storage unit (shown in Fig. 1.1) there are three barriers for the outward movement of activity -

First the borosilicate matrix in which waste oxides are incorporated.

Second the high integrity of all welded stainless steel canister &

Third the steel welded carbon steel secondary containment.

For the purpose of surveillance activity the loss would mean partial or total loss of all the three barriers. This would happen on the highly unlikely event of solidified waste melting down & corroding both the primary & secondary containment. If and when air borne activity is detected in the cooling air, the storage unit which has lost its integrity can be isolated, withdrawn, rescanned and restored to its position.

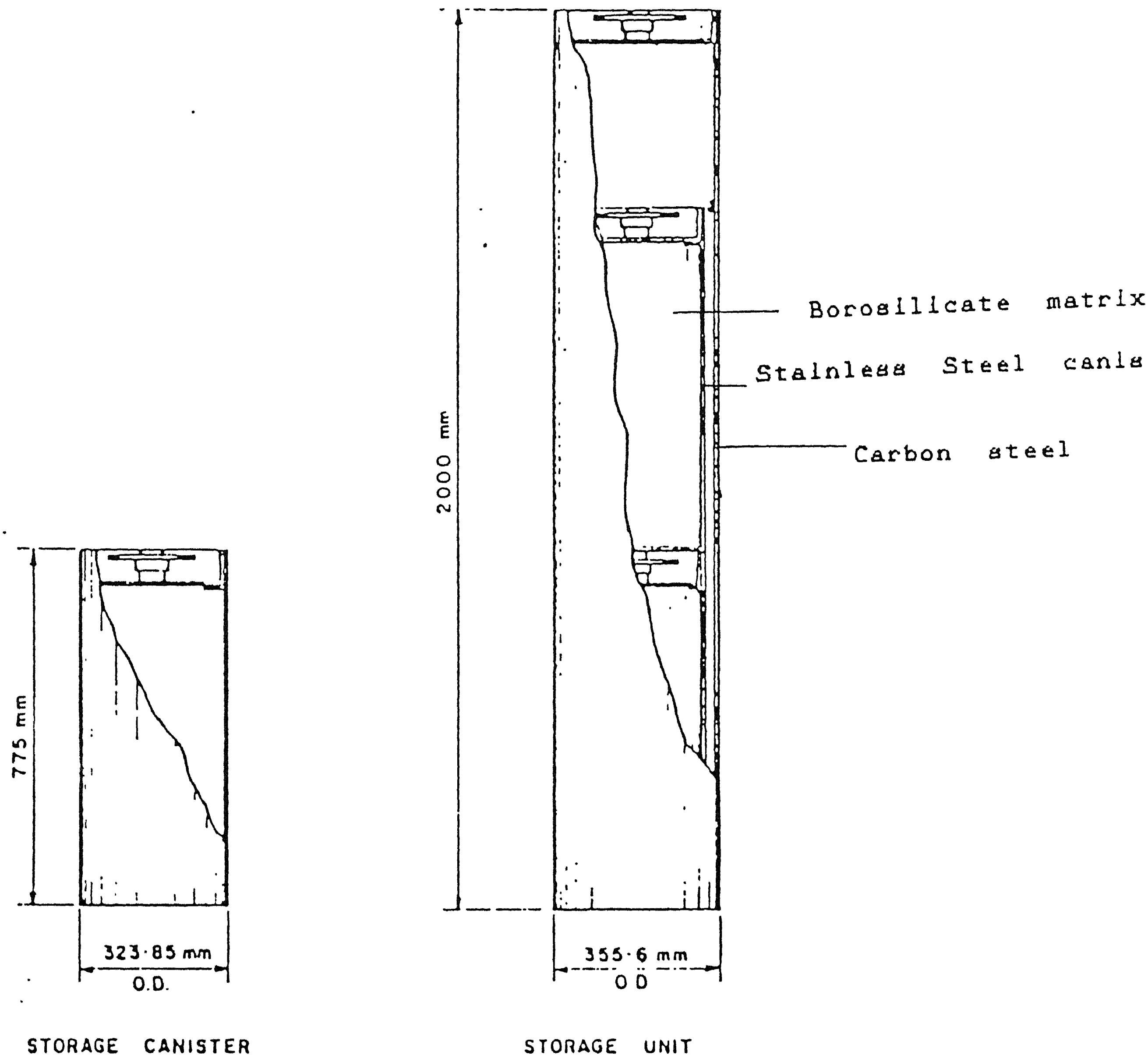


Fig. 1.1 Schematic of storage unit

1.6 NATURAL CONVECTIVE AIR COOLING STORAGE [1]

The air cooled storage is based on natural convection air cooling with induced draught . It utilizes a stack to provide the driving force for air movement through the storage vault . The system is self regulating & can compensate for the changed load . As the decay heat reduces , both the air flow and air temperature decrease .

Storage vault

It is the building (shown in Fig. 1.2) in which storage units are arranged in a definite array , in order to optimize heat transfer efficiency . To facilitate this , storage vault is divided into two blocks and each is further subdivided into three compartments at Tarapur Storage Facility . It is necessary to isolate thermal effects from the load bearing structure and also isolate any water ingress . This is achieved by recourse to a double vault design . The inner storage vault is designed on thermal considerations & the design of external is based on structural and biological shielding considerations as well as protection from all credible accidents .

The filling pattern of the storage vault may be progressive or compartment wise . In progressive filling pattern each compartment receives one storage unit successively . In this pattern the peak heat load in any compartment would not exceed about 510 Kw , with an

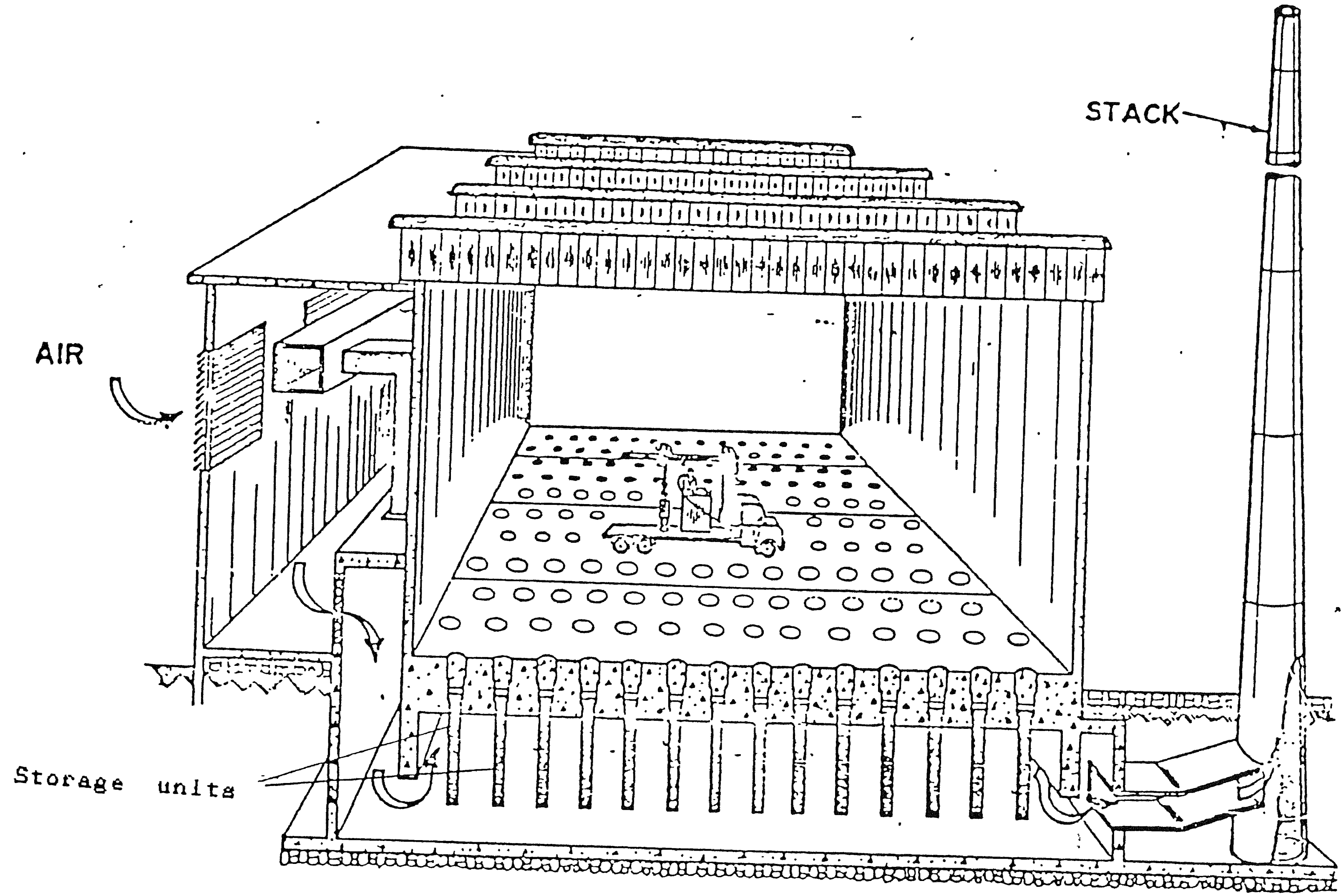


Fig. 1.2 Schematic of air cooled vault

wise filling i.e. if a compartment is to receive one storage unit every alternate day . Then the maximum cumulative heat load would be about 820 Kw and activity load around 0.3 billion curies . While progressive filling has the advantage of providing for lower heat and activity loads in the vault structure , compartment wise filling would provide isolation of radioactive inventory in any one compartment . Each of these filling pattern is further subdivided into two loading patterns namely forward and backward , forward loading means placing the unit in the direction of air flow and backward implies the reverse . Backward has an advantage that the fresh storage unit with maximum heat generation comes in contact with the cooler air while in forward loading the storage unit with maximum heat generation comes in contact with the hotter air .

1.7 OUTLINE OF PRESENT WORK

In this thesis convective heat transfer coefficients have been estimated for different arrangements of storage units . The storage unit may be held vertically or horizantally with free or forced convective air cooling . It may also be held within a sleeve , in which both the convection and radiation heat transfer occur simultaneously .

In the steady state analysis of a storage unit , the two dimensional temperature distribution has been

obtained . The method used is finite difference . From the computer programme prepared the temperature variation can be obtained in a cicular cylindrical object , radially as well as heightwise , subjected to forced or free convection .

In the transient analysis part the time has also been regarded as a variable . It is very useful from accidental analysis view point , as it gives the results under extremely pessimistic conditions . Here , variation of atmospheric temperature , change in radial temperature distribution etc. with time has been obtained .

Chapter 2

ESTIMATES OF CONVECTIVE AND RADIATIVE HEAT TRANSFER COEFFICIENTS

For the estimation of convection heat transfer coefficient (hc) different empirical relations have been used depending upon the type geometry and flow conditions . The various configurations are discussed one by one . Throughout the medium is air .

2.1 CALCULATIONS OF FLOW RATE AND PRESSURE LOSS THROUGH THE TUBE BANKS [6,9]

Storage units are held vertically in a triangular pitch of 825 × 825mm.Hence the arrangement is staggered tube flow . (Fig. 2.1)

Given

Air inlet temperature $T_{\infty 1} = 25\ ^0C.$

Maximum permissible air outlet temperature $T_{\infty 2} = 85\ ^0C.$

Total surface area for heat transfer is $A = N.\pi.d.l$

where ,

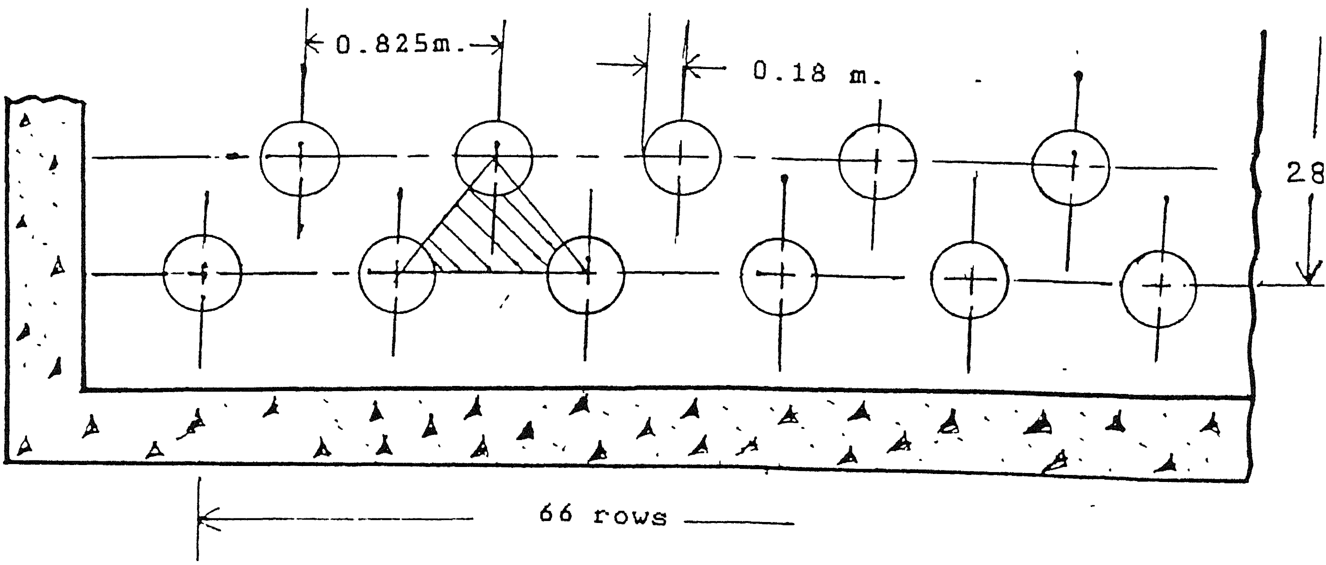
N = Total number of storage units = 1848 units

d = diameter of each unit = 35.56 cm.

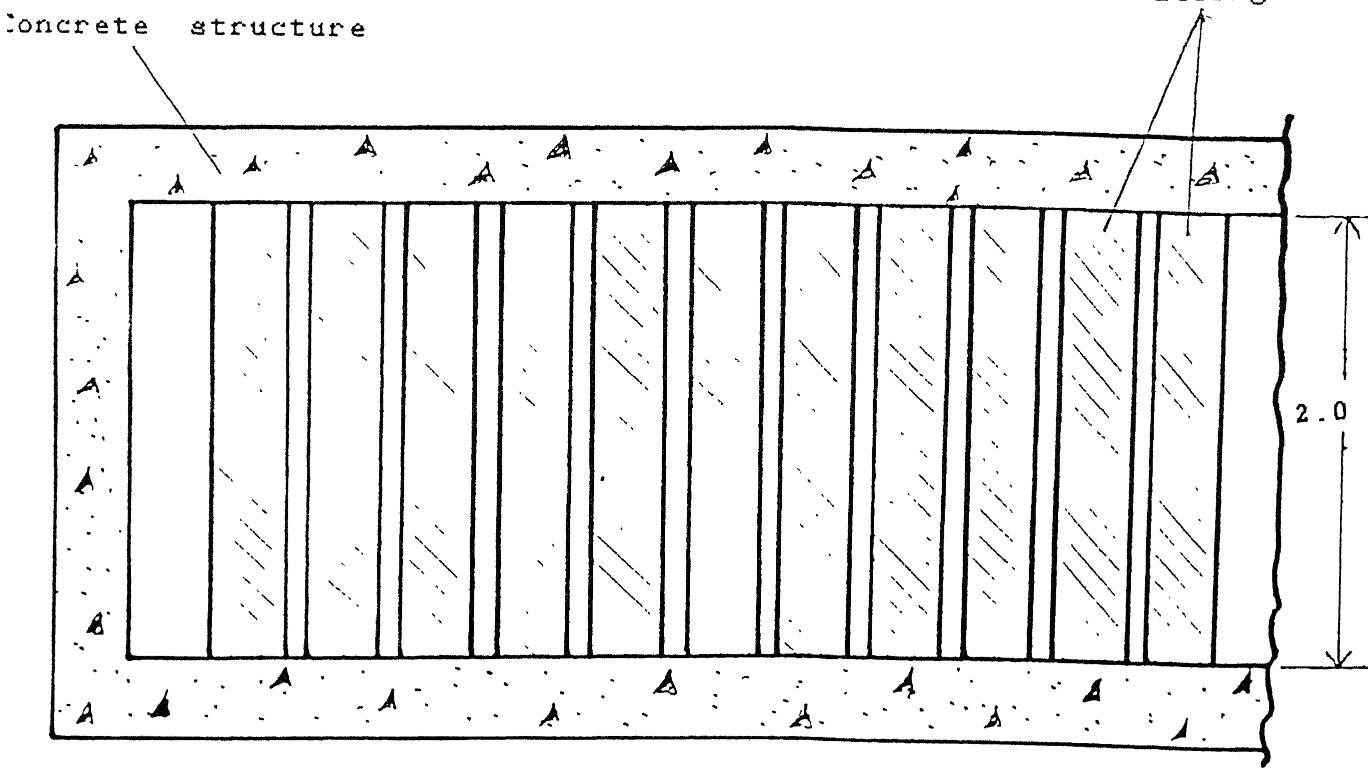
l = length of each unit = 200 cm.

hence

$A = 1848 \times \pi \times 35.56 \times 200$



Top view



Front view

Fig. 2.1 Arrangement of storage units in the storage vault

$= 4.13 \times 10^7$ Cm.^{2}

$= 4130$ meter^2

From the heat balance equation

$$Q = \dot{m} \times C_p \times (T_{\infty 2} - T_{\infty 1}) \qquad (2.1)$$

where ,

Q = Total heat load = 4000 kW (However actual load is 3060 kW, but for design purpose 4000 KW has been taken)

C_p = Specific heat of air = 1.006 kJ $kg^{-1}K^{-1}$.

$\dot{m}$ = Mass flow rate , kg s^{-1}

hence

$$\dot{m} = \frac{4000}{1.006 \times (85 - 25)}$$

This gives

$$\dot{m} = 66.282 \text{ kg s}^{-1}$$

Density of air -

$$\rho_\infty = \frac{P}{R\ T_\infty} \qquad \text{-----(From the gas law)}$$

where ,

ρ_∞ = Density of the atmospheric air

P = Pressure of the atmospheric air

$= 1.0132 \times 10^5$ N m^{-2}

R = Gas constant for air = 287 J $kg^{-1}K^{-1}$

T_∞ = Temperature of the atmospheric air = 298 K.

hence

$$\rho_\infty = 1.0132 \times 10^5/(287 \times 298)$$

$= 1.185$ kg m^{-3}

Therefore volume flow rate of air is

$$V = \dot{m}/\rho_\infty$$

$= 56.92\ m^3\ s^{-1}$

$= 2,01,324\ m^3\ hr^{-1}$

For pressure loss calculation The formula used is -

$$\Delta P = \frac{2 \times f \times G_{max}^{2} \times N_t}{\rho} \times (\mu_w/\mu_b)^{0.4} \qquad (2.2)$$

where ,

G_{max} = mass velocity at the minimum area , kg $m^{-2}s^{-1}$

ρ = density evaluated at the free stream condition

μ_b = average free stream viscosity , Pa. s

μ_w = viscosity of air at the wall temperature (179^0C.),

f = friction factor

N_t = number of transverse rows = 66

For the given arrangement

$$f = \left[0.25 + \frac{0.118}{[(S_n - d)/d]^{1.08}} \right] \times Re_{max}^{-0.16} \qquad (2.3)$$

where ,

Re_{max} = Reynolds number at the minimum flow area

The minimum flow area is

$A_{min} = (S_n - d) \times l \times N_g$

For the given arrangement

S_n = 825 mm.

d = Diameter of the storage unit = 35.56 cm.

l = length of the unit = 200.0 cm.

N_g = number of gaps for the whole storage vault

for the given arrangement N_g = 32

hence

$A_{min} = (82.5 - 35.56) \times 200 \times 32 \times 10^{-4}\ m^2$

$= 30.0\ m^2$

And mass velocity at the minimum area

$$G_{max} = \dot{m}/A_{min}$$
$$= 2.21 \ Kg/m^2 sec$$

flow velocity at the minimum area

$$V_{max} = V/A_{min}$$
$$= 56.92/30.0$$
$$= 1.897 \ m \ s^{-1}$$

Also

$$\rho_{\infty} = 1.185$$
$$\mu_b = 2 \times 10^{-5} \ Pa.\ s \quad (at \ 25 \ ^0C.)$$
$$\mu_w = 2.4 \times 10^{-5} \ Pa.\ s \quad (at \ 179 \ ^0C.)$$

hence

$$Re_{max} = (\rho_{\infty} \times V_{max} \times d)/\mu_b$$
$$= (1.185 \times 1.897 \times 0.3556)/ 2 \times 10^{-5}$$
$$= 39968$$

And hence friction factor is

$$f = \left[0.25 + \frac{0.118}{[(82.5-35.56)/35.56]^{1.08}} \right] \times (39968)^{-0.16} \quad (2.4)$$

$$= 0.062$$

Therefore

$$\Delta p = \frac{(2 \times 0.062 \times 2.21 \times 66)}{1.185} \times \left(\frac{2.4}{2.0} \right)^{0.4}$$

$$= 15.66 \ N \ m^{-2}$$
$$= 1.596 \ kg \ m^{-2}$$
$$= 1.50 \ meter \ of \ air$$

This pressure loss is very small in comparison to head developed by the stack , which is around 100 meters

of air hence it does not affect the stack height much .

2.2 CONVECTION HEAT TRANSFER COEFFICIENT FOR FLOW ACROSS STORAGE UNITS [6]

The formula to be used for the estimation of average heat transfer coefficient for flow across the tube bundles is (shown in Fig. 2.1)

$$\frac{h\ d}{K_f} = C \left[\frac{U_{\infty}\ d\ \rho_f}{\mu_f}\right]^n Pr^{1/3} \qquad (2.5)$$

$$= C\ (Re_{max})^n\ Pr^{1/3}$$

The units are held in a triangular pitch and there is flow across them hence the above formula will be used to calculate the heat transfer coefficient

Where ,

h = Convection heat transfer coefficient , $W\ m^{-2}K^{-1}$

d = Tube diameter , m.

K_f = Thermal conductivity of air at film temperature, $W\ m^{-1}K^{-1}$

U_{∞} = Free stream velocity of air , $m\ s^{-1}$

Pr = Prandtl number at the film temperature

ρ_f = Density at the film temperature , $kg\ m^{-3}$

μ_f = Dynamic viscosity at the film temperature , Pa. s

c and n are constants for a particular arrangement .

For the given arrangement

C = 0.523

$n = 0.56$

The film temperature is

$$T_f = (T_w + T_\infty) / 2$$

where ,

T_w = Temperature of storage unit wall = 179 ^{0}C.

T_∞ = Temperature of the surrounding air = $\dfrac{T_{in} + T_{out}}{2}$

hence

$T_\infty = (25+85)/2 = 55\ ^0C.$

$T_f = 117\ ^0C. = 390\ K$

At this temperature

$\mu_f = 2.4 \times 10^{-5}\ kg\ m^{-1}s^{-1}$

$K_f = 0.03\ W\ m^{-1}K^{-1}$

$Pr = 0.80$

and

$Re_{max} = 39968$ (same as in section 2.1)

hence

$$h\ \frac{35.56}{100}\ \frac{1}{0.03} = 0.523 \times (39968)^{0.56} \times (0.8)^{1/3}$$

this gives

$h = 15.46\ W/m^2\ ^0C.$

This is the convection heat transfer coefficient for flow across the storage units .

2.3 FREE CONVECTION HEAT TRANSFER FROM HORIZONTAL AND VERTICAL CYLINDERS [6,7]

Horizontal cylinder

The correlation for free convection heat transfer

from a horizontal cylinder is (shown in Fig. 2.2)

$$Nu^{1/2} = 0.60 + 0.387 \times \left[\frac{Gr\ Pr}{[1 + (0.559/Pr)^{9/16}]^{16/9}} \right]^{1/6} \qquad (2.6)$$

for $10^{-5} < Gr\ Pr < 10^{12}$

where ,

Nu = Nusselt number

Pr = Prandtl number

Gr = Grashof number

All properties are evaluated at the film temperature .

Given

diameter of cylinder	d	= 0.36 meter
length of cylinder	l	= 2.0 meter
Ambient temperature of air	T_∞	= 30^0C.
Rate of heat generation in the cylinder	q'''	= 12.96 KW/m^3

here h is calculated by trial and error method

<u>assume</u> $h = 10\ W/m^2K$, heat balance gives

$$q''' \times \frac{\pi}{4} \times d^2\ l = h \times \pi\ d\ l \times (T_w - T_\infty) \qquad (2.7)$$

where all the terms are same as before

this gives $T_w = T_\infty + \frac{q'''.\ d}{4\ h}$

Putting numerical values this gives

$T_w = 145.2^0$ C.

therefore

$T_f = (T_w + T_\infty)/2 = 87.6^0$C. = 360.6 K

At this temperature

ν = Kinematic viscosity of air = $21.85 \times 10^{-6}\ m^2\ s^{-1}$

$K_f = 0.03\ W\ m^{-1}\ K^{-1}$

$Pr = 0.691$ and $Gr\ Pr = 2.04 \times 10^8$ (which is in the given range)

Putting these values in the above correlation

$Nu = 70.17$

or

$h\ d/K = 70.17$

putting d and K into this it gives

$h = 6.12\ W\ m^{-2} K^{-1}$

This value is much different than the assumed one hence repeating above calculations with this h

Now

$T_f = 124.13^0\ C. = 397.13\ K$

$Pr = 0.686$

$\nu = 25.92 \times 10^{-6}\ m^2\ s^{-1}$

$K = 0.034\ W\ m^{-1} K^{-1}$

$Gr\ Pr = 2.14 \times 10^8$

and putting these values into above correlation we get

$Nu = 71.14$

This gives

$h = 6.74\ W\ m^{-2} K^{-1}$

Which is quite near to the last value , hence the value of convection heat transfer coefficient for horizontal cylinder is $6.74\ W\ m^{-2} K^{-1}$.

Vertical cylinder

For vertical cylinder with same dimension as for horizontal cylinder the correlation used is (shown in Fig. 2.3)

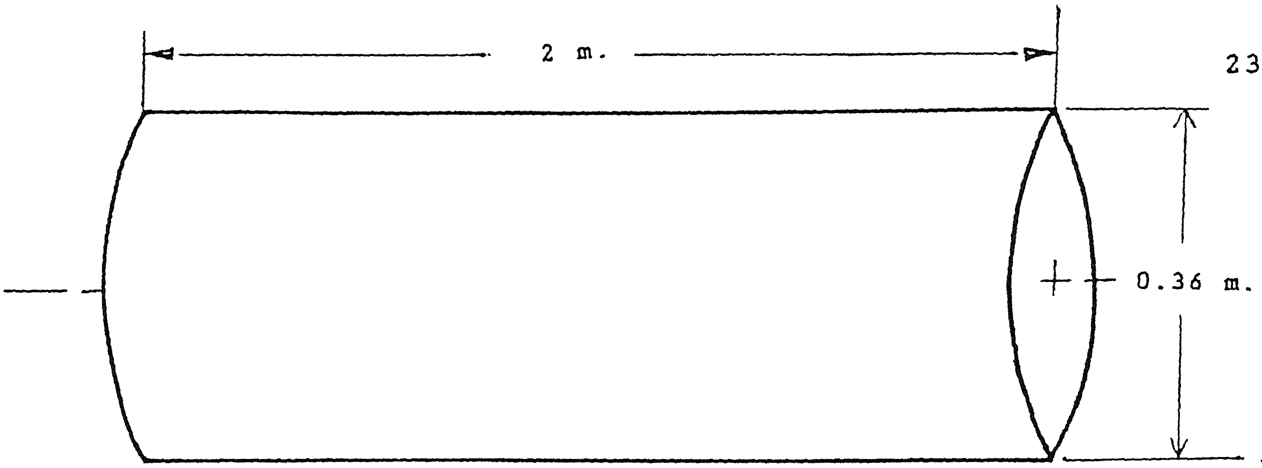


Fig. 2.2 Free convection heat transfer from a horizantal cylinder

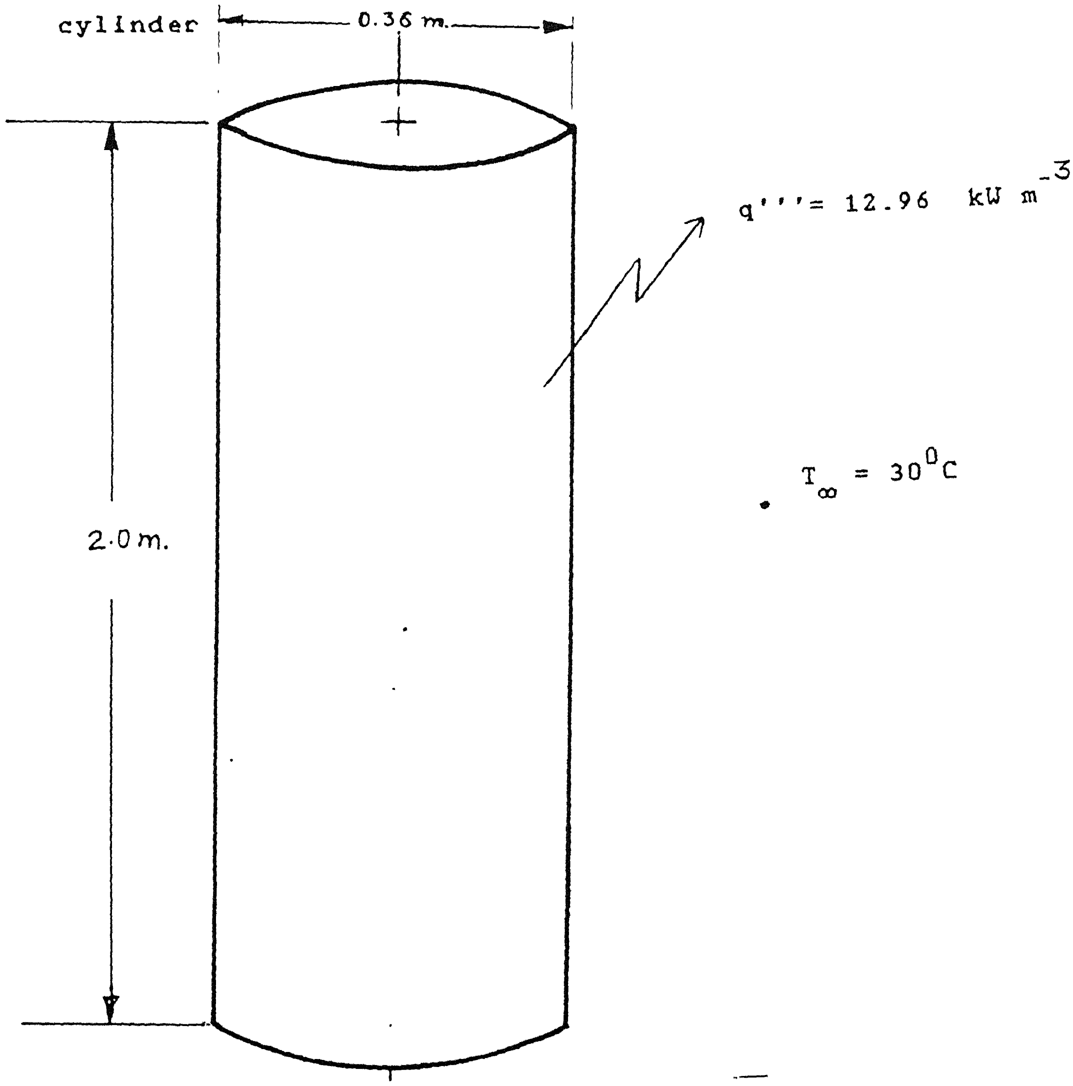


Fig. 2.3 Free convection heat transfer from a vertical cylinder

$$Gr_x Nu_x = \frac{g\ \beta\ q''x^4}{K\ \nu^2} = Gr_x^* \qquad (2.8)$$

For laminar range :

$Nu_x = 0.60 \times (Gr_x^* \times Pr)^{1/5}$ for $10^5 < Gr_x^* < 10^{11}$

and

$h_{av.} = 5/4 \times h_{x=1}$

For turbulent range :

$Nu_x = 0.17 \times (Gr_x^* \times Pr)^{1/4}$ for $2\times10^{13} < Gr_x^* < 10^{16}$

and

$h_{av.} = h_{x=1}$

where ,

Gr_x = Grashof number at x

Nu_x = Nusselt number at x

Gr_x^* = Modified Grashof number at x

β = $1/T_f$

K = Thermal conductivity of the air at T_f

μ = Dynamic viscosity of air

x = Distance

ere again trial and error method will be used

ssume $h = 10\ W/m^2.K$

nd

$q''' \frac{\pi}{4} d^2 l = h \times \pi d l \times (T_w - T_\infty)$ gives

$T_f = 87.6^0C. = 360.6$ K (as was done for horizontal ylinder).

: this temperature properties of air are

$\nu = 21.85 \times 10^{-6}\ m.^2 s^{-1}$

$K = 0.03\ W\ m^{-1}K^{-1}$

$Pr = 0.691$

and

$\beta = 1/T_f = 2.77 \times 10^{-3} K^{-1}$

Putting these values in above correlations

$Gr_x^* = 1.84 \times 10^{12} x^4$

Now up to $Gr_x^* = 10^{11}$ the flow is laminar

i.e. up to $x = (10^4 / 1.84 \times 10^{12})^{1/4} = 0.48$ m.

$= 48.0$ cm.

and after this the flow is turbulent

Now with this Gr_x^*

For laminar range $h_x = 4.9 / x^{0.2}$ $W/m^2.K$

and for turbulent range $h_x = 6.14$ $W/m^2.K$

Now for the whole range the average h is

$$h_{av.} = \frac{\int_0^{0.48} h_x|_{laminar}\, dx + \int_{0.48}^{1.0} h_x|_{turbulent}\, dx}{1}$$

$= 3.40 + 3.20$

$= 6.60 \; W\, m^{-2} K^{-1}$

This value of h is quite different from the assumed one hence repeating above calculations with this h

$T_f = 117.3^0C. = 390.3$ K

$\nu = 25.45 \times 10^{-6} \; m^2 s^{-1}$

$K = 0.034 \; W\, m^{-1} K^{-1}$

$Pr = 0.686$

$Gr_x^* = 1.15 \times 10^{12} x^4$

Now flow is laminar up to $x = 54.0$ cm.

for laminar range $h_x = 4.83/x^{0.2}$

& for turbulent range $h_x = 5.38$

on averaging above two $h_{av.} = 6.20\ W\ m^{-2} K^{-1}$

Again repeating above calculations with new h we get

$h_{av.} = 6.17\ W\ m^{-2} K^{-1}$

This value is quite close to the previous one hence for vertical cylinder the convection heat transfer coefficient is $6.17\ W\ m^{-2} K^{-1}$.

2.4 FREE CONVECTION HEAT TRANSFER THROUGH A VERTICAL ANNULAR SPACE

The empirical correlation used for this case (shown in Fig. 2.4) is

$$Nu_{\delta} = 0.42.(Gr_{\delta}.Pr)^{1/4}.Pr^{0.012}.(l/\delta)^{-0.3} \tag{2.9}$$

where ,

δ = annular gap = 5 cm.

and all other terms have usual meaning , all the properties are evaluated at the film temperature . Here again trial and error method will be used

<u>assume</u> $h = 10\ W\ m^{-2} K^{-1}$, this gives (from the heat balance equation used in section 2.3)

$T_w = 145.2\,^0C.$ and

$T_f = 87.6\,^0C. = 360.6\ K$

The Grashof number is given by

$$Gr_{\delta} = \frac{g\beta(T_w - T_{\infty})\delta^3}{\nu^2} \tag{2.10}$$

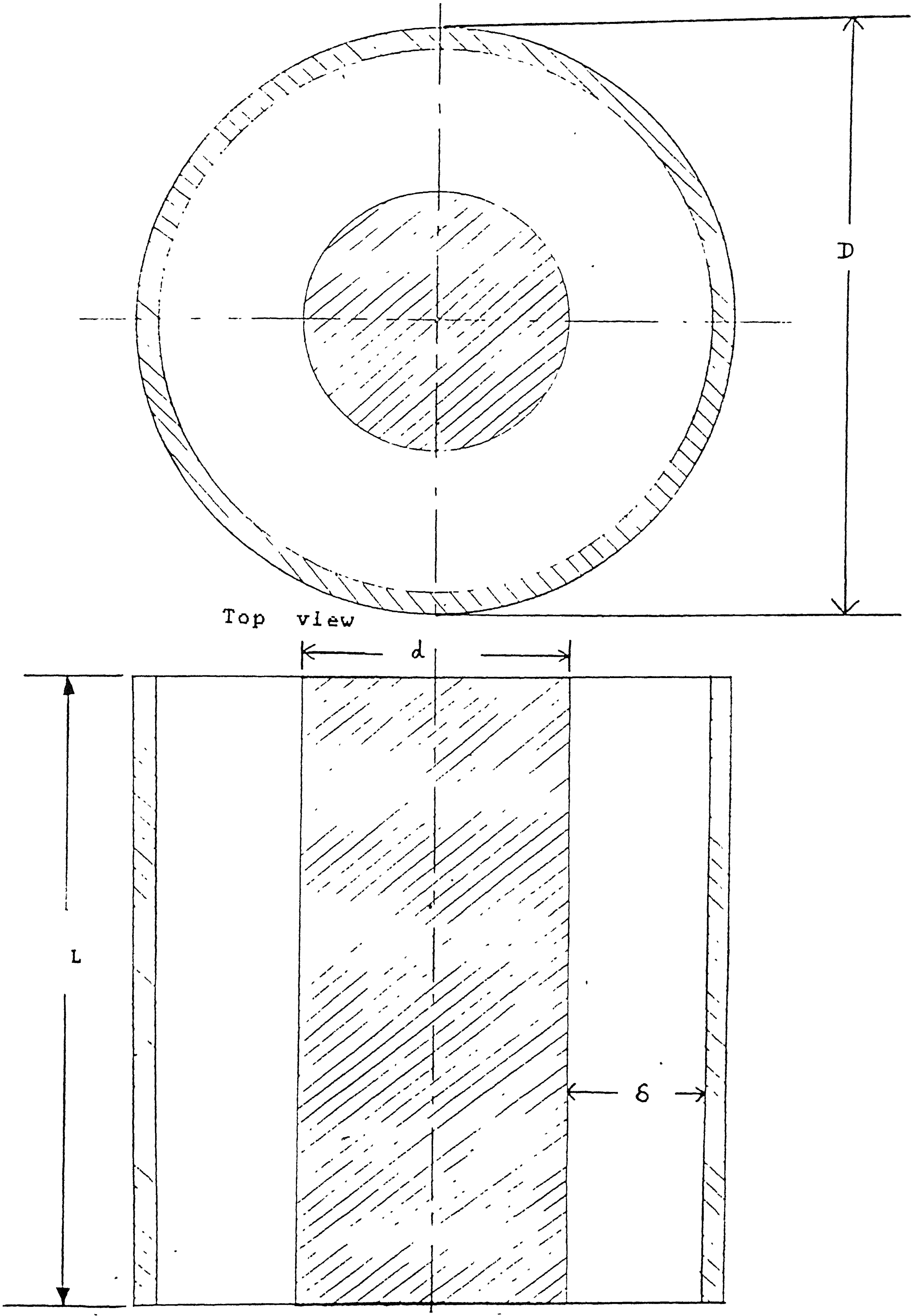


Fig. 2.4 Free convection heat transfer through a vertical annular space

where all the terms are same as before .

Evaluating properties of the air at T_f and putting into above we get

$Gr_\delta = 819612.1$

and

$Gr_\delta . Pr = 819612.1 \times 0.691 = 566352$

Putting these values in the above correlation

$Nu_\delta = 3.80 = h\delta / K$

hence

$h = 2.35 \ W \ m^{-2} K^{-1}$

Now with this h repeating above calculations we get

$T_f = 275.1^0C. = 548.1 \ K$

$Gr_\delta = 553322.1$

$pr = 0.676$

and

$Nu_\delta = 3.42$

which gives

$h = 3.0 \ W \ m^{-2} K^{-1}$

Again doing the same thing with this new h

$Gr_\delta = 668171.6$

$Pr = 0.683$

$Nu_\delta = 3.59$

this gives

$h = 2.94 \ W \ m^{-2} K^{-1}$

This value is quite near to the previous one hence for vertical annular space $h = 2.94 \ W \ m^{-2} K^{-1}$

2.5 RADIATION HEAT TRANSFER THROUGH A VERTICAL ANNULAR SPACE [6]

An alternative arrangement in the storage vault may be such that the storage units are held in a vertical annular space , and there is a cross flow of air through the banks of storage units . After this the air is made to pass through annular space of individual units . In this way there will be two convection heat transfers -

- First due to cross flow of air through the storage units, it occurs at the outer surface of the sleeve .
- Second due to free convection of air through the annular space between the storage unit and its sleeve .

In addition to these there is a radiation heat transfer between the sleeve , storage unit and surrounding . Hence the total heat transfer coefficient (h_{total}) in this case is

$$h_{total} = h_c + h_r \tag{2.11}$$

where ,

h_c = Free convection heat transfer coefficient for the annular space

h_r = Radiation heat transfer coefficient for the storage unit

h_c has already been found in the previous section .In order to get h_r , radiation analysis of the annular space (shown in Fig. 2.5) is carried out . The case is treated as, one surface held inside an enclosure

with a window in it .

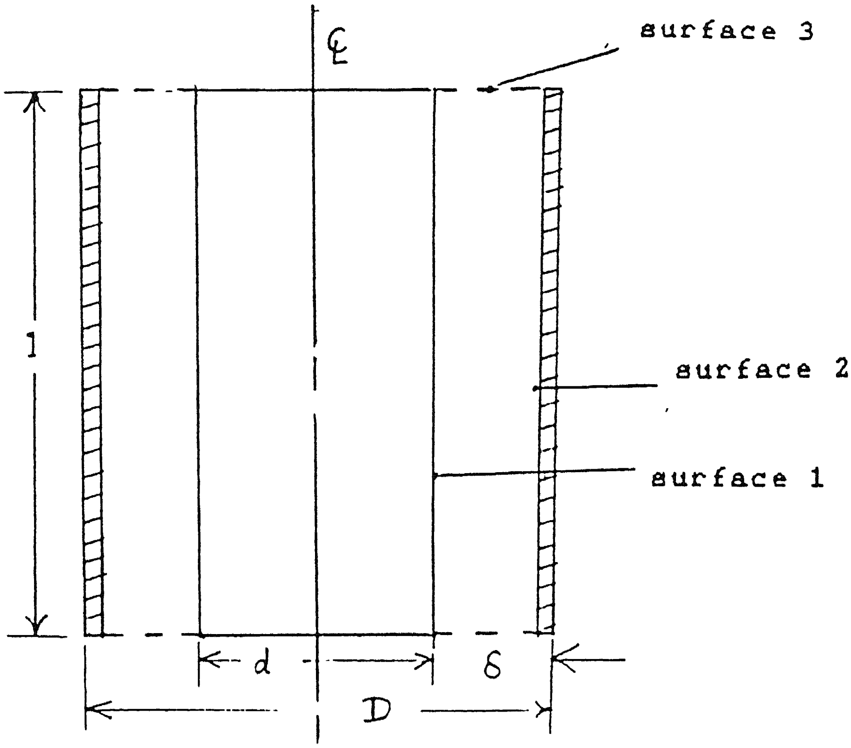


Fig. 2.5 Radiation analysis of annular space

Given

d = diameter of the cylinder = 0.36 m

l = length of the cylinder = 2.0 m

D = diameter of the sleeve = 0.46 m

δ = annular gap = 0.05 m

q_{gen} = rate of heat generation = 2164.5 W

emmissivities of the three surfaces are

$\epsilon_1 = 0.6$

$\epsilon_2 = 0.6$

$\epsilon_3 = 1.0$

areas of the three surfaces are

$A_1 = \pi . d . l = 2.26 \ m^2$

$A_2 = \pi . D . l = 2.89 \ m^2$

$A_3 = \frac{\pi}{4} (D^2 - d^2) = 0.064 \ m^2$

now the radiation network for the above arrangement is shown in Fig. 2.6

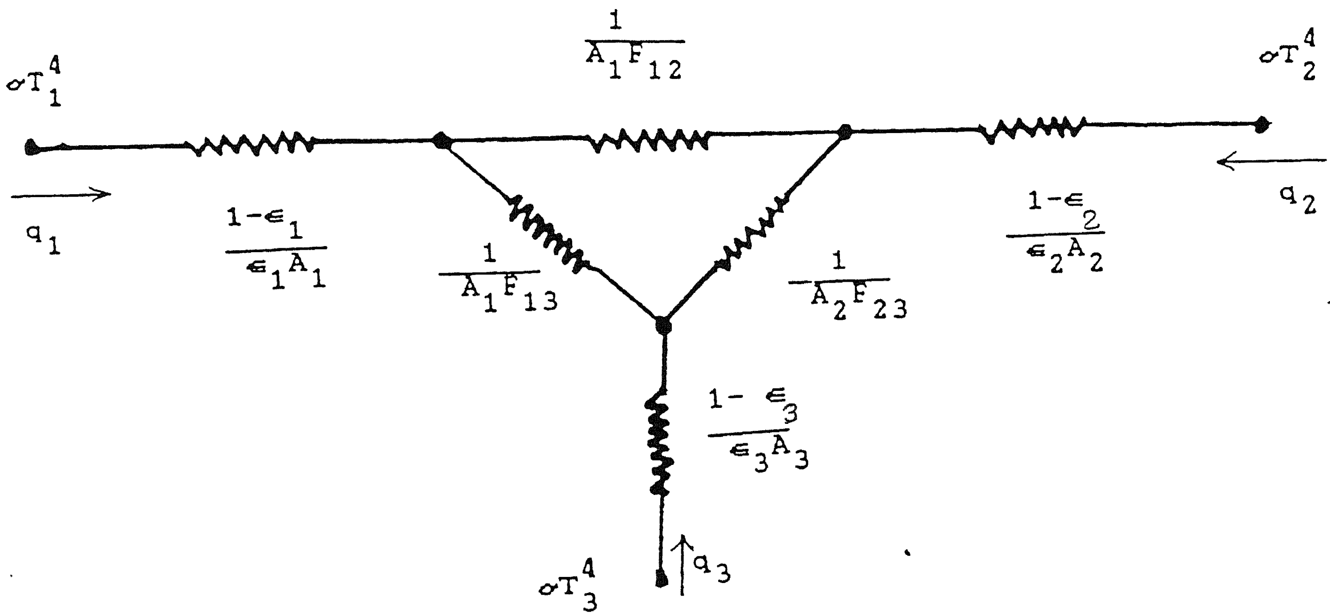


Fig. 2.6 Radiation network

where,

q_1 = net heat loss from surface 1

q_2 = net heat loss from surface 2

q_3 = net heat loss from surface 3

T_1 = absolute temperature of surface 1

T_2 = absolute temperature of surface 2

T_3 = absolute temperature of surface 3 = 303 K

ϵ_1 , ϵ_2 and ϵ_3 are the emmissivities of three surfaces

F_{ij} = shape factor from surface i to surface j

σ = Stefan - Boltzmann constant = $5.669 \times 10^{-8} \ W \ m^{-2} K^{-4}$

Shape factors F_{21} and F_{22} for the given dimensions

are

$$F_{21} \cong o.78$$

$$F_{22} \cong 0.22$$

from the reciprocity relation

$$A_1 \ F_{12} = A_2 \ F_{21} \quad , \text{ this gives}$$

$$F_{12} = \frac{A_2}{A_1} F_{21}$$

putting numerical values this gives

$$F_{12} \simeq 1.0$$

now since

$$F_{21} + F_{22} + F_{23} = 1.0$$

putting numerical values into it we get

$$F_{23} \simeq 0.0$$

Therefore final network is shown in Fig. 2.7

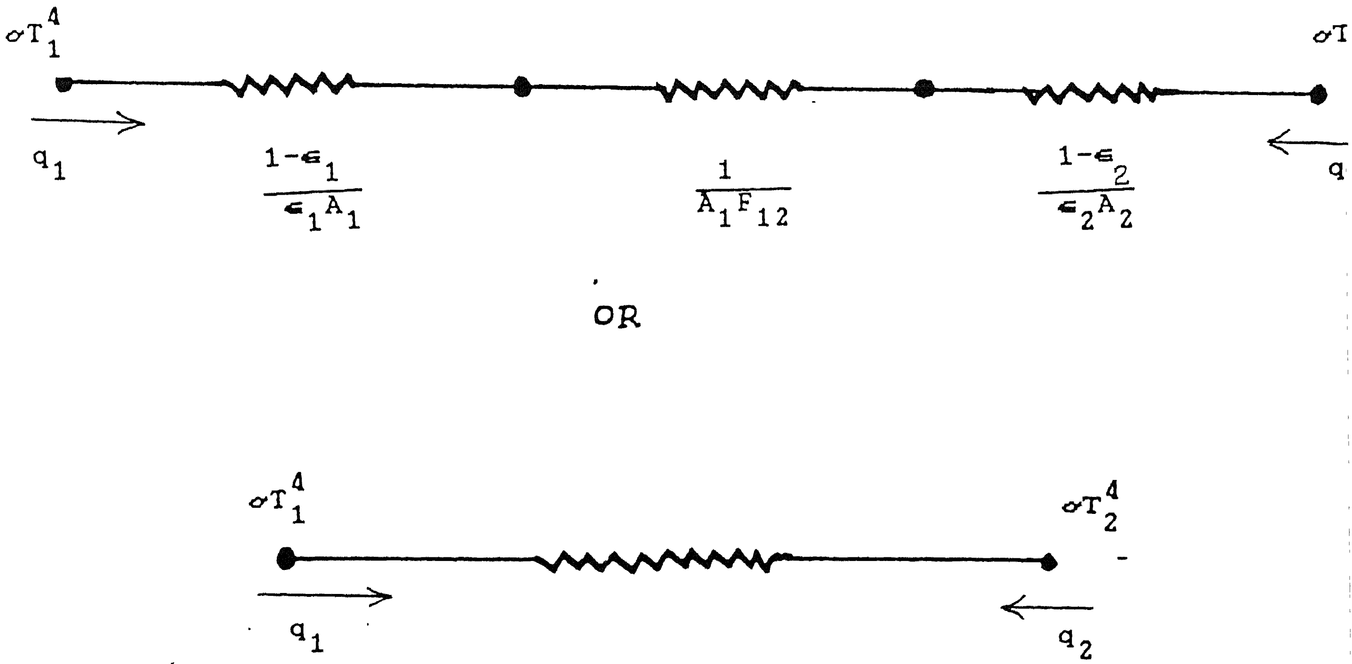


Fig. 2.7 Final network

in this network

$$\frac{1-\epsilon_1}{\epsilon_1 A_1} = 0.295$$

$$\frac{1}{A_1 F_{12}} = 0.440$$

$$\frac{1-\epsilon_2}{\epsilon_2 A_2} = 0.231$$

now from the radiation network

$$q_1 + q_2 = 0$$

and

$$q_1 = \frac{\sigma\,(T_1^4 - T_2^4)}{0.966} \qquad (2.12)$$

The energy balance for the cylinder and sleeve is

$$q_{gen} = q_1|_{rad} + h_c|_{in} . A_1 . (T_1 - T_f) \qquad (2.13)$$

and

$$h_c|_{in} . A_2 . (T_f - T_2) + q_1|_{rad} = h_c|_{out} . A_2 . (T_2 - T_3) \qquad (2.14)$$

where ,

T_f = film temperature = $\frac{T_1+T_2}{2}$

$h_c|_{in}$= convection heat transfer coefficient for the surface 1 and the inside surface of sleeve

= 2.94 W $m^{-2}K^{-1}$ (calculated in section 2.4

$h_c|_{out}$= convection heat transfer coefficient for the outside surface of sleeve

= 15.64 W $m^{-2}K^{-1}$ (calculated in section 2.2)

q_{gen} = heat generation rate in a storage unit

= 2164.5 W

$q_1|_{rad}$= net heat loss from surface 1 due to radiation

T_3 = ambient temperature = 30^0C = 303 K

on adding equns. 2.13 and 2.14 and putting the value of T_f

$$q_{gen} + h_c|_{in}.A_2.(T_1-T_2)/2 = h_c|_{in}.A_1.(T_1-T_2) + h_c|_{out}.A_2.(T_2-T_3)/2$$

putting numerical values into it

$$2164.5 + 4.25\ (T_1-T_2) = 3.32\ (T_1- T_2) + 45.2\ (T_2-303)$$

on simplifying this

$$T_2 = \frac{15860+0.93\ T_1}{46.13} \qquad (2.15)$$

now put the values of q_{gen} , $q_1|_{rad}$ and T_2 into eqn. 2.13

$$\sigma \times [\ T_1^4 - 0.0113\ T_1^3 - 287.1\ T_1^2 - 3268345\ T_1 - 1.39 \times 10^{10}\ ]/0.966$$
$$+ \ 3.25\ T_1 - 1141.45 = 2164.5$$

putting $\sigma = 5.669 \times 10^{-8}$ and simplifying above equation -

$$5.87 \times 10^{-8}T_1^4 - 6.63 \times 10^{-10}T_1^3 - 1.685 \times 10^{-5}T_1^2 + 3.06\ T_1 = 4121.7 \qquad \text{---}(2.16)$$

Solving above equation numerically for T_1

T_1 = 463.4 K , and from eqn. 2.15

T_2 = 353.15 K

hence $q_1|_{rad}$= 1732.40 W , and

$$h_r = \frac{1732.40}{A_1.(T_1- T_3)}$$

$h_r = 4.78 \ W\,m^{-2}K^{-1}$

This is the radiation heat transfer coefficient for an annular space , and its value is around two times of that due to convection heat transfer . The value of h_r may vary with the variation of annular gap and emmissivities of the surfaces . These variations are shown in Tables 2.1 and 2.2 .

TABLE 2.1

Variation of heat transfer coefficients with annular gap

ANNULAR GAP (δ), cm	CONVECTION HEAT TRANSFER COEF. (h_c), $Wm^{-2}K^{-1}$	RADIATION HEAT TRANSFER COEF. (h_r), $Wm^{-2}K^{-1}$	TOTAL HEAT TRANSFER COEF. (h_{total}), $Wm^{-2}K^{-1}$
5.00	2.94	4.78	7.72
7.50	2.96	5.06	8.02
10.00	3.00	5.16	8.16

TABLE 2.2

Variation of radiation heat transfer coefficient with emmissivities of surfaces (ϵ_1 and ϵ_2) for a constant δ

EMMISSIVITY $\epsilon_1 = \epsilon_2$	h_r $W\ m^{-2}K^{-1}$
0.5	4.35
0.6	4.78
0.7	5.56

2.6 RADIATION HEAT TRANSFER IN FLOW ACROSS STORAGE UNITS

In this case radiative heat transfer occurs between the storage unit and the surrounding concrete structure .The surface of neighbouring storage unit does not affect the radiation heat transfer , because they are at the same temperature . The units are held in a triangular pitch of 825 × 825 mm. (shown in Fig. 2.1). This case is treated as , one surface completely enclosed inside the other surface .

The network of radiation heat transfer is (shown in Fig. 2.8)

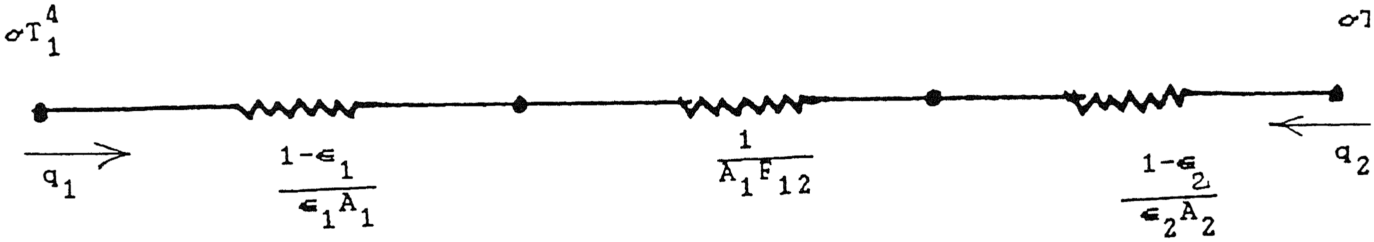


Fig. 2.8 Network of radiation heat transfer

where,

T_1 = temperature of the cylinder surface

= 452 K (from section 2.2)

T_2 = 323 K (assume)

ϵ_1 = emmissivity of surface 1 = 0.6

ϵ_2 = emmissivity of surface 2 = 0.6

A_1 = area of surface 1 = $\pi . d . l$

d = diameter of the cylinder = 0.36 m

l = length of the cylinder = 2.0 m

A_2 = area of surface 2

$$= 2.2 \left[\frac{0.825 \times 0.825 \sqrt{3}}{2 \times 2} - \frac{1}{2} . \frac{\pi}{4} . 0.36 \right]$$

$$= 0.975 \ m^2$$

hence ,

$$\frac{A_1}{A_2} = 2.32$$

now for the network of Fig. 2.8

$$q_1 = \frac{\sigma (T_1^4 - T_2^4)}{\dfrac{1-\epsilon_1}{\epsilon_1 A_1} + \dfrac{1}{A_2 F_{21}} + \dfrac{1-\epsilon_2}{\epsilon_2 A_2}} \qquad (2.17)$$

and the shape factor $F_{21} = 1.0$, since surface 2 sees 1 only hence equn. 2.17 reduces to

$$q_1 = \frac{\sigma . (T_1^4 - T_2^4) . A_1}{\dfrac{1}{\epsilon_1} + \dfrac{A_1}{A_2 \epsilon_2} - 1}$$

putting numerical values into it

$q_1 = 872.0$ W

hence ,

$$h_r = \frac{872}{A_1 . (T_1 - T_2)}$$

$= 3.0$ W $m^{-2}K^{-1}$

This is the radiation heat transfer coefficient , while the h_c for this case was 15.46 W $m^{-2}K^{-1}$, hence the total heat transfer coefficient in this case is 18.46 W $m^{-2}K^{-1}$.

2.6(a) RADIATION HEAT TRANSFER WITH PERFECTLY BLACK SURFACES

From section 2.5 it is clear that in case of annular space the radiation heat transfer coefficient rises rapidly with the increase of emmissivity . Therefore it is possible to achieve a higher heat transfer rate by making the surfaces perfectly black. Under these conditions radiation analysis of both the previous cases is discussed below

ANNULAR SPACE

In this case (shown in Fig. 2.5) all the parameters leaving emmissivities are same as in section 2.5 . The emmissivities in this case are

$\epsilon_1 = 1.0$; $\epsilon_2 = 1.0$; $\epsilon_3 = 1.0$

with these values various resistances of the radiation network (shown in Fig.2.7) become

$$\frac{1-\epsilon_1}{\epsilon_1 A_1} = 0.0 \ ; \quad \frac{1}{A_1 F_{12}} = \frac{1}{A_1} \quad \text{and} \quad \frac{1-\epsilon_2}{\epsilon_2 A_2} = 0.0$$

hence net radiation heat transfer is

$$q_1 = \sigma A_1 \left(T_1^4 - T_2^4 \right) \qquad (2.18)$$

where,

A_1 = Area of surface 1 = 2.26 m^2

Using same procedure as in section 2.5 to evaluate T_1 and T_2 , we get

$$1.29 \times 10^{-7} T_1^4 - 9.2 \times 10^{-9} T_1^3 - 3.7 \times 10^{-5} T_1^2 + 2.83\ T_1 = 5096.84 \qquad \text{----}(2.19)$$

and

$$T_2 = \frac{15860 + 0.93 T_1}{46.13} \qquad (2.20)$$

solving equn. 2.19 numerically and putting the value of T_1 thus obtained into equn. 2.20 , we get

T_1 = 417.6 K = 144.6 ^{0}C

T_2 = 352.2 K = 79.2 ^{0}C

now from equn. 2.18

$q_1|_{rad}$ = 1935.8 W

therefore radiation heat transfer coefficient is

$$h_r = \frac{q_1|_{rad}}{A_1.(T_1-T_3)} \quad ; \quad T_3 = \text{Ambient temperature} = 30^0C$$

$$= 7.47 \ W\ m^{-2}K^{-1}$$

This is around 250% of convection heat transfer coefficient . This shows that radiation contributes a major portion of total heat transfer in this case .

FLOW ACROSS STORAGE UNITS

In this case

$\epsilon_1 = 1.0$; $\epsilon_2 = 1.0$ and all other parameters are same as in section 2.6 . Various resistances of the network of radiation heat transfer (shown in Fig. 2.8) are

$$\frac{1-\epsilon_1}{\epsilon_1 A_1} = 0.0 \quad ; \quad \frac{1-\epsilon_2}{\epsilon_2 A_2} = 0.0 \quad ; \quad \frac{1}{A_2 F_{21}} = \frac{1}{A_2}$$

and hence

$$q_1 = \sigma . A_2 . (T_1^4 - T_2^4) \tag{2.21}$$

putting same numerical values in this as in section 2.6

$$q_1 = 5.669 \times 10^{-8} \times 0.975 (452^4 - 323^4)$$

$$= 1705.5 \ W$$

hence

$$h_r = \frac{1705.5}{A_1.(T_1-T_2)}$$

$$= 5.85 \ W\ m^{-2}K^{-1}$$

This is only about 38% of convection heat transfer coefficient in this case .

Chapter 3

STEADY STATE TEMPERATURE DISTRIBUTION

Here steady state temperature distribution inside a solid circular cylinder , generating heat at a constant rate will be estimated . The method used is Finite Difference Method .

3.1 BASIC EQUATION [6]

The basic heat conduction equation in cylindrical coordinates is :

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'''}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau} \qquad (3.1)$$

Where ,

T = temperature

r = radius

ϕ = base angle

z = height

τ = time

q''' = rate of heat generation

K = thermal conductivity

α = thermal diffusivity

For steady state $\frac{\partial T}{\partial \tau} = 0$

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Where ,

T = temperature

r = radius

ϕ = base angle

z = height

τ = time

q''' = rate of heat generation

K = thermal conductivity

α = thermal diffusivity

For steady state $\frac{\partial T}{\partial \tau} = 0$

also there is no variation along $\emptyset$ hence $\frac{\partial^2 T}{\partial \emptyset^2} = 0$

Therefore final heat conduction equation is :

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{q'''}{K} = 0 \qquad (3.2)$$

Now finite difference method will be used to solve above equation

3.2 FINITE DIFFERENCE METHOD [11,12,13,14]

In this approach , the problem domain is 'discretized' so that the dependent variables are considered to exist only at discrete points . Derivatives are approximated by differences resulting in an algebraic representation of the partial differential equation . The nature of resulting system of algebraic equations depends upon the character of the problem posed by the original partial differential equation .

Here network lines are given by

$r_i = (i-1) \times \Delta r$ and

$z_i = (j-1) \times \Delta z$ $\qquad i , j = 1 , 2 , 3 , \ldots..$

Quantities Δr and Δz are increments in r and z directions respectively . The points of intersection of the network are called nodes . The network and nodes are shown in Fig. 3.1

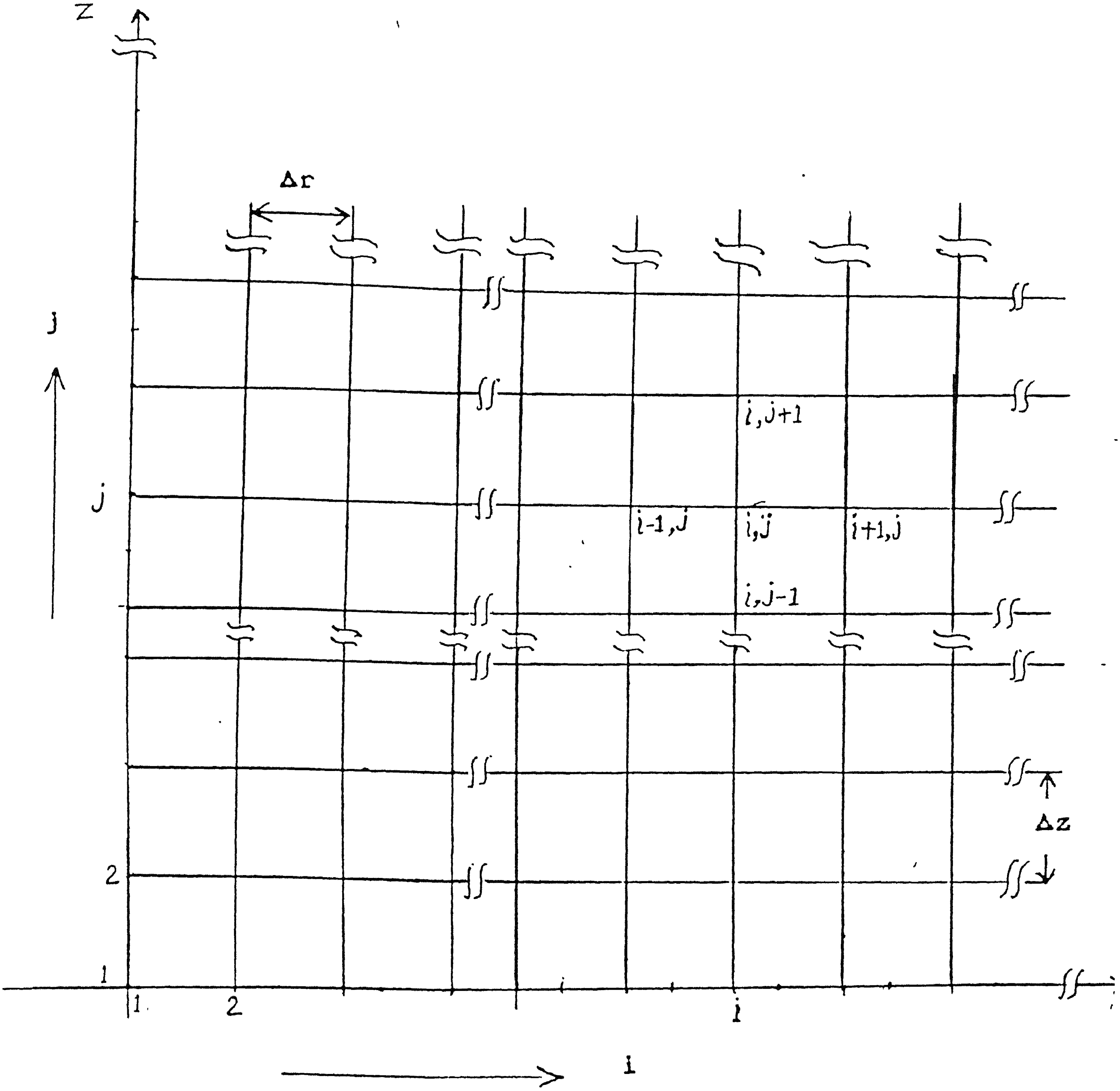

Figure - 3.1 : Descretization of the problem

various difference equations are :

Forward difference _

$$\frac{\partial T}{\partial r} = T_r\Big|_{i+1,j} = \frac{T_{i+1,j} - T_{i,j}}{\Delta r}$$

Backward difference _

$$\frac{\partial T}{\partial r} = T_r\Big|_{i,j} = \frac{T_{i,j} - T_{i-1,j}}{\Delta r}$$

Central difference _

$$\frac{\partial T}{\partial r} = T_r\Big|_{i,j} = \frac{T_{i+1,j} - T_{i-1,j}}{2\Delta r}$$

Out of the above three differences the central difference formula will be used .

$$\frac{\partial^2 T}{\partial r^2} = T_{rr}\Big|_{i,j} = \frac{T_{i-1,j} - 2 \times T_{i,j} + T_{i+1,j}}{\Delta r^2}$$

similarly

$$\frac{\partial^2 T}{\partial z^2} = \frac{T_{i,j-1} - 2 \times T_{i,j} + T_{i,j+1}}{\Delta z^2} = T_{zz}\Big|_{i,j}$$

Putting these expressions in equation 3·2 and simplifying

$$\frac{h^2}{k^2} (T_{i,j+1} + T_{i,j\_1}) = 2 \times T_{i,j} (\frac{h^2}{k^2} + 1) - T_{i-1,j} (1 - \frac{1}{2(i-1)}) - T_{i+1,j} (1 + \frac{1}{2(i-1)}) - h^2 C \quad . \qquad (3.3)$$

Where , $C = \frac{q'''}{K}$, $h = \Delta r$, $K = \Delta z$

. equation 3.3 which involves five node points , is evaluated at each of the node points of network formed in this way n linear algebraic equations are formed if there are n number of node points in the network . In

each of these equations only five out of n terms are non zero . while evaluating these equations following boundary conditions are used -(shown in Fig. 3.2)

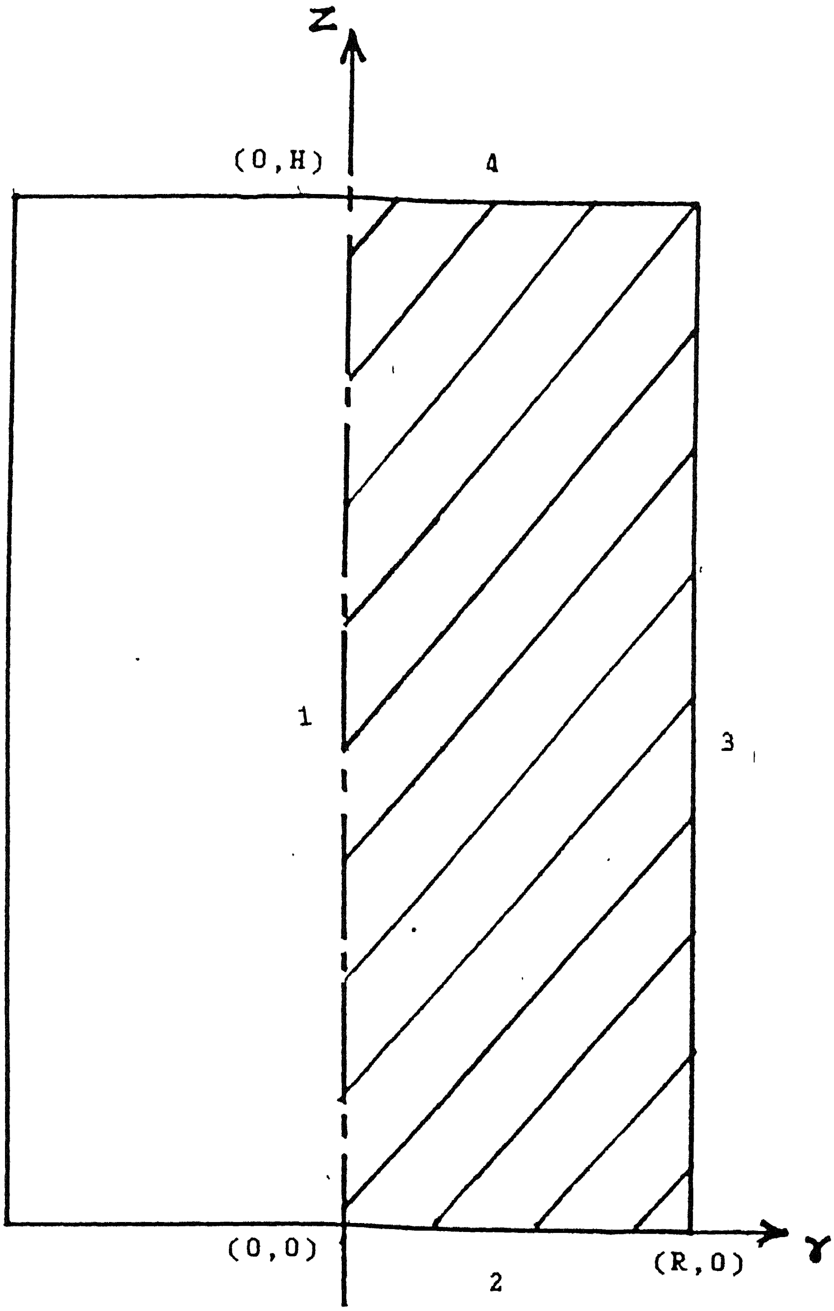

Fig. 3.2 Boundries at which boundry conditions are taken

For boundary no. 1 (at $r = 0$)

$$\frac{\partial T}{\partial r} = 0 \qquad \text{i.e.} \qquad T_{1,j} = T_{2,j} \tag{3.4}$$

For boundary no. 2 (at $z = 0$)

$$- K \times \frac{\partial T}{\partial z} = h \times (T_{i,1} - T_{amb.}) \tag{3.5}$$

or

$$- K \times \frac{T_{i,2} - T_{i,1}}{\Delta z} = h \times (T_{i,1} - T_{amb.})$$

For boundary no. 3 (at $r = R$)

$$- K \times \frac{\partial T}{\partial r} = h \times (T_{n+1,j} - T_{amb.}) \tag{3.6}$$

or

$$- K \times \frac{T_{n+1,j} - T_{n,j}}{\Delta r} = h \times (T_{n+1,j} - T_{amb.})$$

For boundary no. 4 (at $z = H$)

$$- K \times \frac{\partial T}{\partial z} = h \times (T_{i,n+1} - T_{abm.}) \tag{3.7}$$

or

$$- K \times \frac{T_{i,n+1} - T_{i,n}}{\Delta z} = h \times (T_{n+1,j} - T_{abm.})$$

where ,

$T_{amb.}$ = Ambient temperature

In this way there will be n linear equations involving n unknown temperatures , these will be solved to obtain the temperatures . The methods of solutions are described below .

3.3 METHODS OF SOLUTION [11,14]

There are numerous methods of solving n linear equations in n unknowns . These can be written in matrix form as

$$A \cdot X = B \qquad (3.8)$$

Where, matrix A is the coefficient matrix

matrix X is the column matrix of unknown temperatures , and matrix B is the column matrix whose elements are the right hand members of linear equations .

Methods of solutions can be classified as direct or iterative . Direct methods are those that provide the solution in a finite and predetermined number of operations using an algorithm that is usually complicated . Iterative methods consist of repeated application of an algorithm that is usually relatively simple . They yield an answer only as a limit of a sequence . The number of iterations required to obtain solutions that no longer change to a specified number of decimal places usually cannot be determined in advance .

3.3.1 DIRECT METHODS

<u>cramer's rule</u>

This is one of the most elementary methods introduced early in the study of algebra . Unfortunately ,

the number of operations required in the algorithm is approximately (n+1)! , for n unknowns . For more than about three equations the use of cramer's rule becomes impractical and is not recommended .

Gaussian elimination

It is a very useful and efficient tool for solving many system of algebraic equations . Although it is one of the earliest methods proposed for solving simultaneous linear equations , it remains among the most important algorithms in use today .

The solution vector remains unchanged if the equations are multiplied or divided by a constant and if any equation is replaced by the sum or difference of that equation and any other equation . These facts are utilized in the elimination strategy .

In Gaussian elimination , the objective is to transform the system into an upper triangular array by eliminating unknowns through algebraic operations . All the non zero coefficients will then be on or above the main diagonal . At that point the n th equation has only one unknown; the (n-1) th equation , only two unknowns; etc . Back substitution , starting with the n th equation , readily provides the solution . The technique consists of two phases : the elimination of the unknowns and back - substitution to obtain the solution . Loss of accuracy can occur in the algorithm due to round off errors . The use

of partial pivoting to avoid division by zero and to improve accuracy will be included as part of the algorithm . If the elements of the coefficient matrix vary greatly in size , it is likely that accuracy can be further improved by the use of scaling .

Elimination Method for Tridigonal Matrices

Gaussian elimination is a general procedure that , in principle , will provide a solution , if one exist , for any system of linear algebraic equations . For systems whose coefficient matrix contains a specific pattern of zeroes , the elimination scheme can be simplified to avoid pointless operations on the zero elements . Such specialized system of algebraic equations occur very frequently in the numerical solutions of partial differential equation . The most common of these are tridiagonal and pentadiagonal , corresponding to equations that contain three or five unknowns respectively .

Other direct methods

Direct methods for solving certain system of algebraic equations that are significantly faster than Guassian elimination do exist . But none of them is completely general . That is , they are applicable only to the algebraic equations arising from a special class of difference equations and associated boundary conditions .

3.3.2 ITERATIVE METHODS [14]

Iterative methods are particularly useful for system in which roundoff error may be a problem in that such schemes , when convergent , can be continued until changes in the solution have been reduced to some prescribed tolerance . Furthermore , iterative procedures can easily take advantage of the sparse nature of the coefficient matrix that are associated with many of the physical problems .(A matrix is sparse when a high percentage of the entries are zero .) On the other hand , iterative procedures are certain to converge only for systems having "diagonal dominance" . Fortunately , many sets of linear algebraic equations originating from physical systems exhibit this diagonal dominance .

Iterative methods can be further broken down into point (or explicit) iterative methods and block (or implicit) iterative methods . For point iterative methods , the same explicit algorithm is applied iteratively to compute the unknown functions , whereas in block iterative methods , subgroups of unknowns are singled out for the solution by elimination schemes in an overall iterative procedure . The most common applications of the block iterative method are special classes of equations arising from the numerical solutions of elliptic partial differential equations .

<u>Gauss Siedel iteration</u>

It is perhaps the simplest of the commonly used iterative methods . When it can be used , the procedure for a general system of equations is to

1) make initial guesses for all unknowns;

2) solve each equation for the unknown whose coefficient is largest in magnitude , using guessed values initially and the most recently computed values thereafter for the other unknowns in each equation ;

3) repeat the solution of the equations in this manner until changes in the unknowns become smaller than prescribed tolerance .

Actually a careful check on step 1 reveals that a guess for the unknowns having the largest coefficient in the first equation is not required . It should be noted that in carrying out step 2 , it is necessary to determine a different unknown from each equation . In most instances , when the Gauss siedel iteration converges , a different unknown can be found to have the coefficient largest in magnitude in each equation . When this does not occur , step 2 should be modified to permit all the unknowns to be determined from the system of equations .

A more formal statement of the algorithm can be given if the equations are first ordered , if possible , so that the coefficient largest in each row is on the main diagonal . Then , if the diagonal elements are all nonzero (

it must be possible to avoid a zero on the main diagonal if a unique solution exists) , and initial guesses for the unknowns have been made ,

$$x_i = \frac{1}{a_{ii}} \left[C_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} \cdot x_j \right] , \qquad i = 1 ,2 , 3 ,\ldots, n$$

where it is understood that the most recently computed values of x are used on the right hand side ; that is , the variables are continuously updated . When old values of x are used on the right hand side until all n variables have been computed , the iteration procedure is known as Jacobi iteration .In the Jacobi procedure , the variables are finally updated before the algorithm is repeated for another iteration . The Gauss Siedel algorithm converges more readily and should always be used .

3.4 TEMPERATURE DISTRIBUTION

The method used here is Gauss - Siedel iteration method for solving the linear algebraic equations .
Following are the dimensions of the storage unit

Length l = 2.0 meter

Diameter d = 0.36 meter

In fact these are the dimensions of the storage unit of Tarapur Waste storage Facility , so that results obtained can be verified later .

other parameters used in the computer programme are
Convection heat transfer coefficient $h = 10\ W\ m^{-2} K^{-1}$

Rate of heat generation in the cylinder $q''' = 12.96\ kW\ m^{-3}$

Thermal conductivity of the material $K = 1\ W\ m^{-1}K^{-1}$

Ambient temperature $T_\infty = 60^0C.$

Length of cylinder $l = 2.0\ m$

Diameter of cylinder $d = 0.36\ m$

In fact the value of convection heat transfer coefficient may be changed in the computer programme in this part calculations have been done using four values of h in order to get a wide range of results .

For the verification of temperature distribution obtained by finite difference method following equations are used :

$$dq = h \times dA \times (T_{surface} - T_{ambient})$$

integrating

$$q = \int h \times (T_{surface} - T_{ambient}) \times dA = q''' \times volume$$

this gives the total heat generated within the cylinder .

Where ,

q = Total heat generated in the cylinder

$dA = 2 \times \pi \times R \times dz$

R is the cylinder radius

dz is elemental height

Hence

$$q = \int_0^H h \times 2 \times \pi \times R \times (T_{surface} - T_{ambient}) \times dz \qquad ----(3.9)$$

To evaluate above integral over the whole surface of storage unit Simpson's 1/3 Rule is used . In this

method Z is discretized into finite number of increments, each of size Δz such that $H = n \times \Delta z$. The rule can be stated as follows :

$$\int_{z1}^{z1+n.\Delta z} f(z) \; dz = \frac{\Delta z}{3} \times [\, f_1 + 4f_2 + 2f_3 + 4f_4 + \ldots + 4f_n + f_{n+1} \,] \qquad \text{----(3.10)}$$

Where $f_n = f\,(\, z1 + (n-1)\,\Delta z)$

From above it obvious that an even number of intervals (odd number of points) are required for the application of Simpson's 1/3 Rule . The smaller we choose Δz more accurate are the results .

In equation (3.9) $T_{surface}$ is the function of height of the storage unit . Its values at each of the node points are obtained from the steady state temperature distribution obtained previously . Finally a computer programme has been prepared to evaluate above integral . The integral evaluates the value of heat generated and this value is compared with the original value of ($q''' \times$ volume) the heat generated in a storage unit . These values should match with each other . However due to numerical calculations there is a small difference between these values in the results obtained by computer programme.

The variation of temperature with height and diameter are shown in Tables 3.1 and 3.2 . Graphically these variation are shown in Fig. 3.3 and 3.4 .

TABLE 3.1

Variation of temperature with radius at half the height

RADIUS m.	TEMPERATURE (deg. cent.)			
	$h = 5Wm^{-2}K^{-1}$	$h = 10Wm^{-2}K^{-1}$	$h = 15Wm^{-2}K^{-1}$	$h = 20Wm^{-2}K^{-1}$
0.00	389.40	289.12	255.62	238.86
0.01	389.40	289.12	255.62	238.86
0.03	387.28	287.01	253.51	236.75
0.04	383.16	282.87	249.37	232.61
0.05	377.15	276.86	243.36	226.60
0.06	369.44	269.14	235.64	218.88
0.08	360.25	259.94	226.43	209.67
0.09	349.80	249.48	215.97	199.21
0.10	338.34	238.01	204.50	187.73
0.12	326.11	225.76	192.25	175.49
0.13	313.35	212.99	179.47	162.71
0.14	300.28	199.90	166.38	149.61
0.15	287.09	186.70	153.18	136.41
0.17	273.96	173.56	140.03	123.26
0.18	261.04	160.62	127.09	110.32

TABLE 3.2

Variation of temperature with height at zero radius

HEIGHT m.	TEMPERATURE (deg. cent.)			
	$h = 5Wm^{-2}K^{-1}$	$h = 10Wm^{-2}K^{-1}$	$h = 15Wm^{-2}K^{-1}$	$h = 20Wm^{-2}K^{-1}$
0.00	211.37	133.50	108.33	95.95
0.14	319.50	238.51	211.91	198.67
0.29	362.46	273.08	243.05	227.94
0.43	379.23	284.12	252.06	235.94
0.57	385.73	287.61	254.64	238.10
0.71	388.24	288.71	255.37	238.68
0.86	389.16	289.05	255.58	238.83
1.00	389.40	289.12	255.62	238.86
1.14	389.16	289.05	255.58	238.83
1.29	388.24	288.71	255.37	238.68
1.43	385.73	287.61	254.64	238.10
1.57	379.23	284.12	252.06	235.94
1.71	362.46	273.08	243.05	227.94
1.86	319.50	238.51	211.91	198.67
2.00	211.37	133.50	108.33	95.95

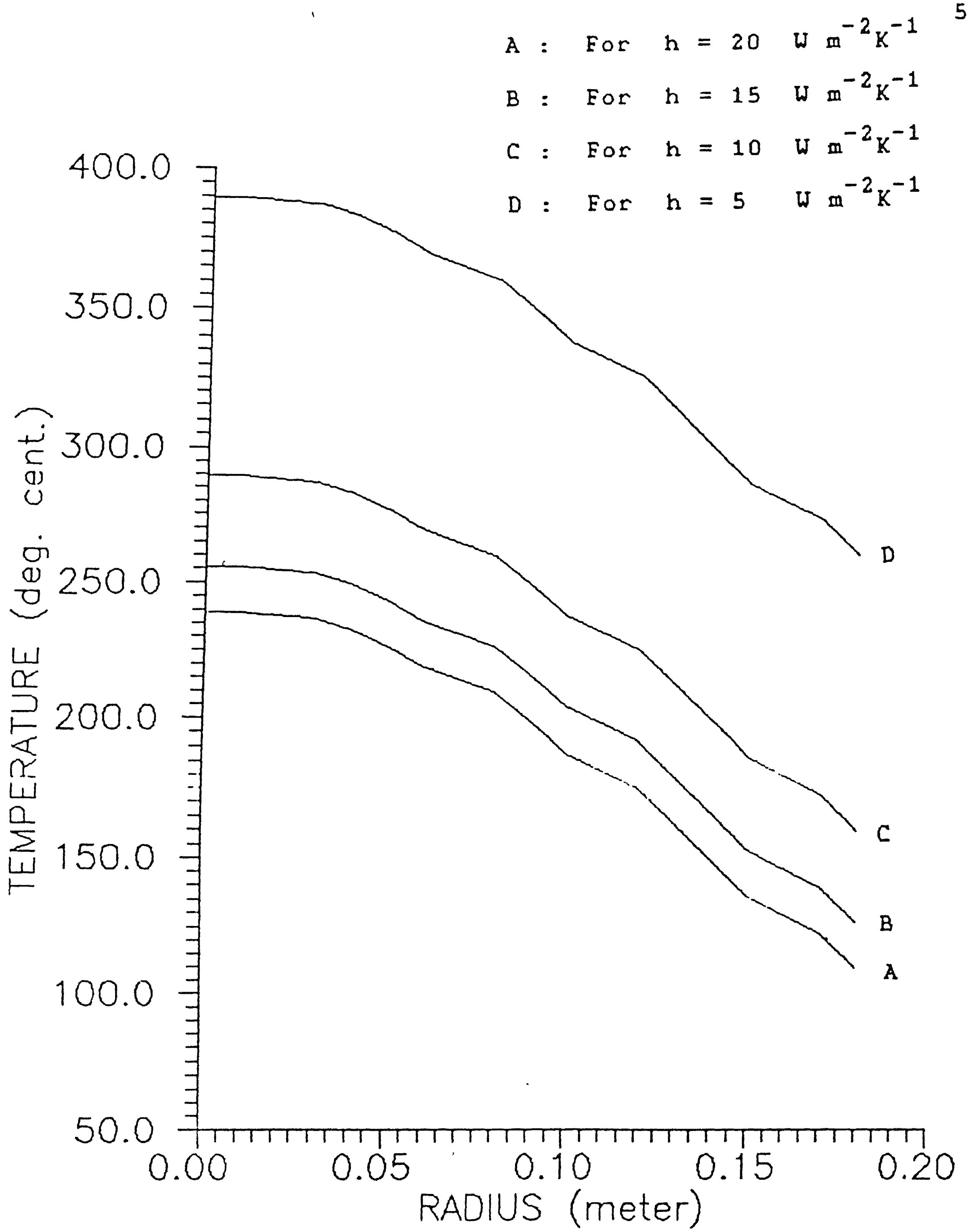

Fig. 3.3 Variation of temperature with radius for different heat transfer coefficients

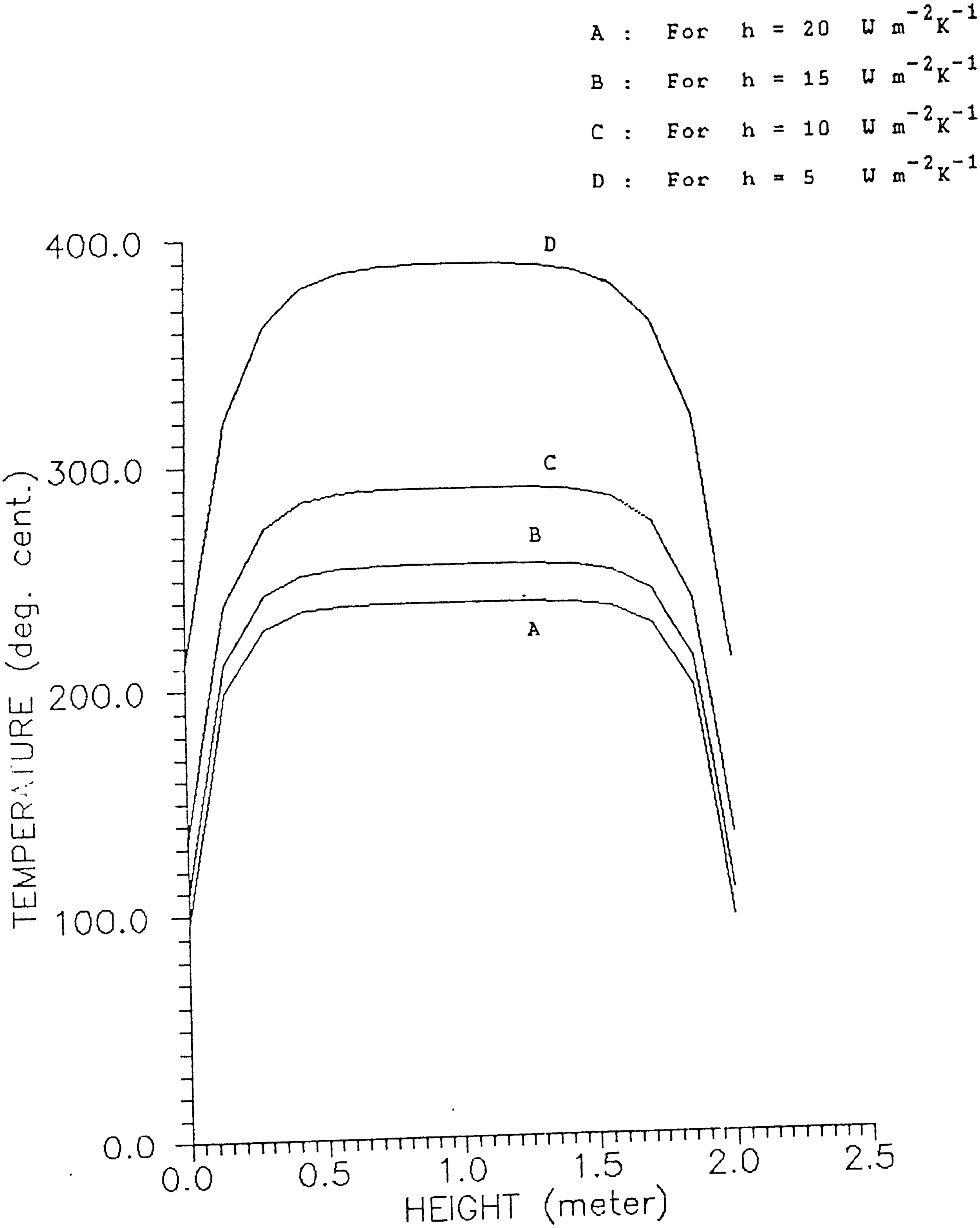

Fig. 3.4 Variattion of temperature with height for different heat transfer coefficents

Chapter 4

TRANSIENT ANALYSIS

4.1 INTRODUCTION

The design of storage vault utilises the decay heat of the waste and engineered design of the vault and the stack induces air flow by natural draught to maintain permissible temperatures . Since this type of storage utilises passive cooling by naturally induced draught to the concrete wall structure , its effectiveness is certain . The maximum credible accident which could severely affect the cooling is one in which the stack collapses as itself and blocks the exhaust plenum , thus blocking the air passage . Such conditions could presumably continue till the plenum is unblocked . A theoretical analysis of the maximum credible accident has been carried out . It should be noted that this analysis is based on a highly pessimistic approach .It has been assumed that both the exit and entrance plenums of the vault are blocked . Further the contribution of the vault walls to the dissipation of heat to the soil has not been considered . The actual temperatures are expected to be much lower than projected .

Provisions have also been incorporated for forced

draught circulation of air employing centrifugal blowers in case of failure of the natural draught cooling , the probability of which is very remote . Arrangement for circulation of air in the annular space between primary and secondary vault are also provided .

To estimate the consequences of such accidental conditions transient analysis of a storage unit is carried out . In this the initial conditions are same as those reached in steady state analysis . The use of this analysis is to get the time after which critical temperatures are reached . Curves have been plotted showing the rate of temperature rise .

4.2 METHOD OF ANALYSIS

The basic equation of transient heat transfer is

$$\frac{1}{\alpha}\frac{\partial T}{\partial \tau} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{q'''}{K} \tag{4.1}$$

where ,

α = Thermal diffusivity

τ = Time

T = Temperature

r = Radius

z = height

q'''= Rate of heat generation

K = thermal conductivity of the storage unit material

Again finite difference scheme will be used to

$T^{t+1}_{ambient}$ = Ambient air temperature at time $t + \Delta t$

$T^{t}_{i,j}$ = Temperature at node (i,j) at time t

$T^{t+1}_{i,j}$ = Temperature at node (i,j) at time $t + \Delta t$

The value of $T^{t+1}_{i,j}$, obtained from (4.2) for every node point is put into (4.3) Which gives the value of $T^{t+1}_{ambient}$. Which is the temperature of ambient air after time Δt. Because of a large number of node points involved in the network a computer programme is prepared. Which gives the temperature after any time T.

4.3 RESULTS

Above calculations are done for three values of h the convection heat transfer coefficient

1. $h = 10 \ W \ m^{-2} K^{-1}$
2. $h = 5 \ W \ m^{-2} K^{-1}$
3. $h = 0.0$

to get a wide range of results. Third one is the worst case in which there is no convection heat transfer between the storage unit and surrounding air. The variation of these temperatures with time are shown in Figures 4.1, 4.2 and 4.3 .

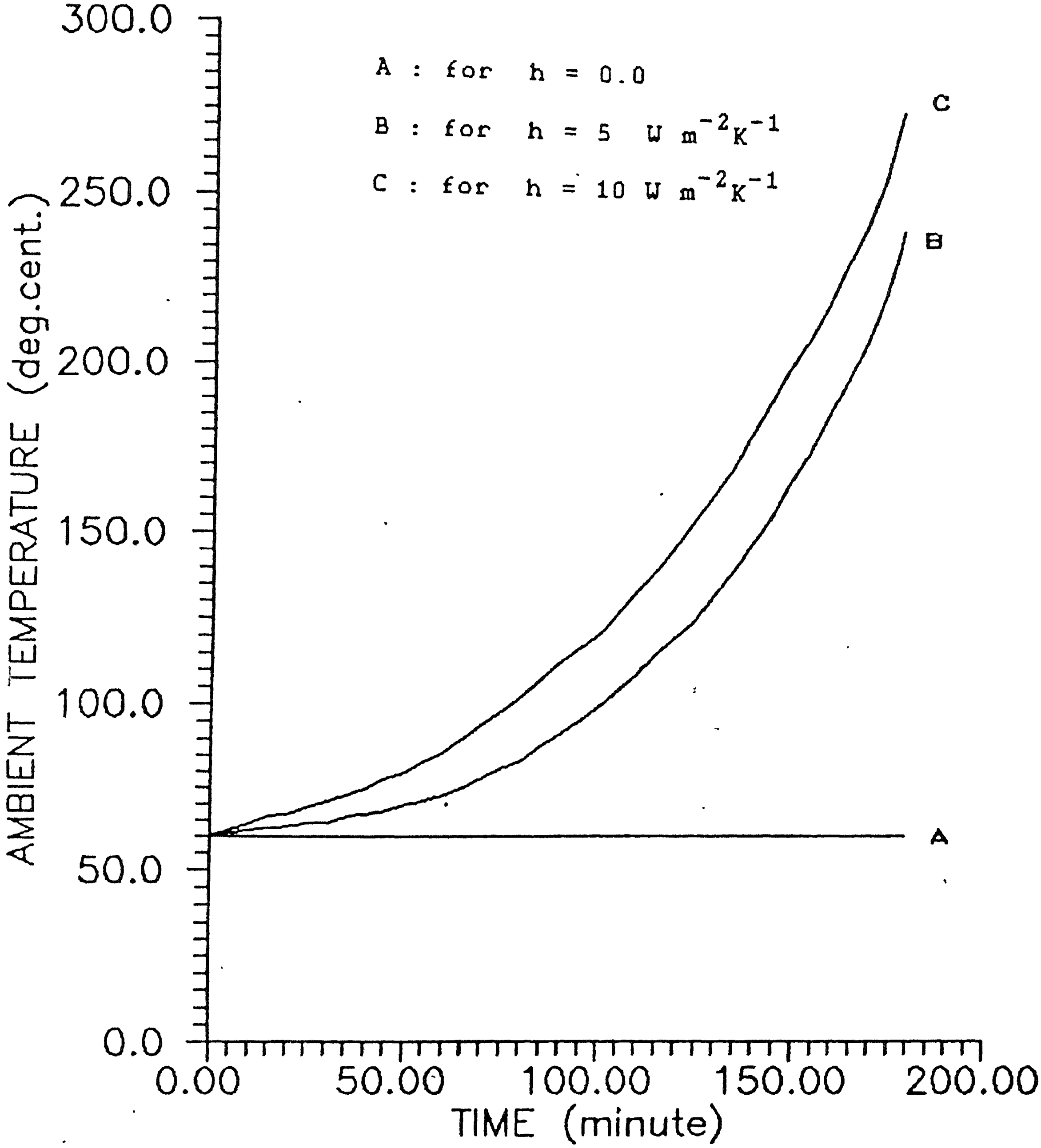

Fig. 4.1 Variation of atmospheric temperature with time and convection heat transfer coefficient (h)

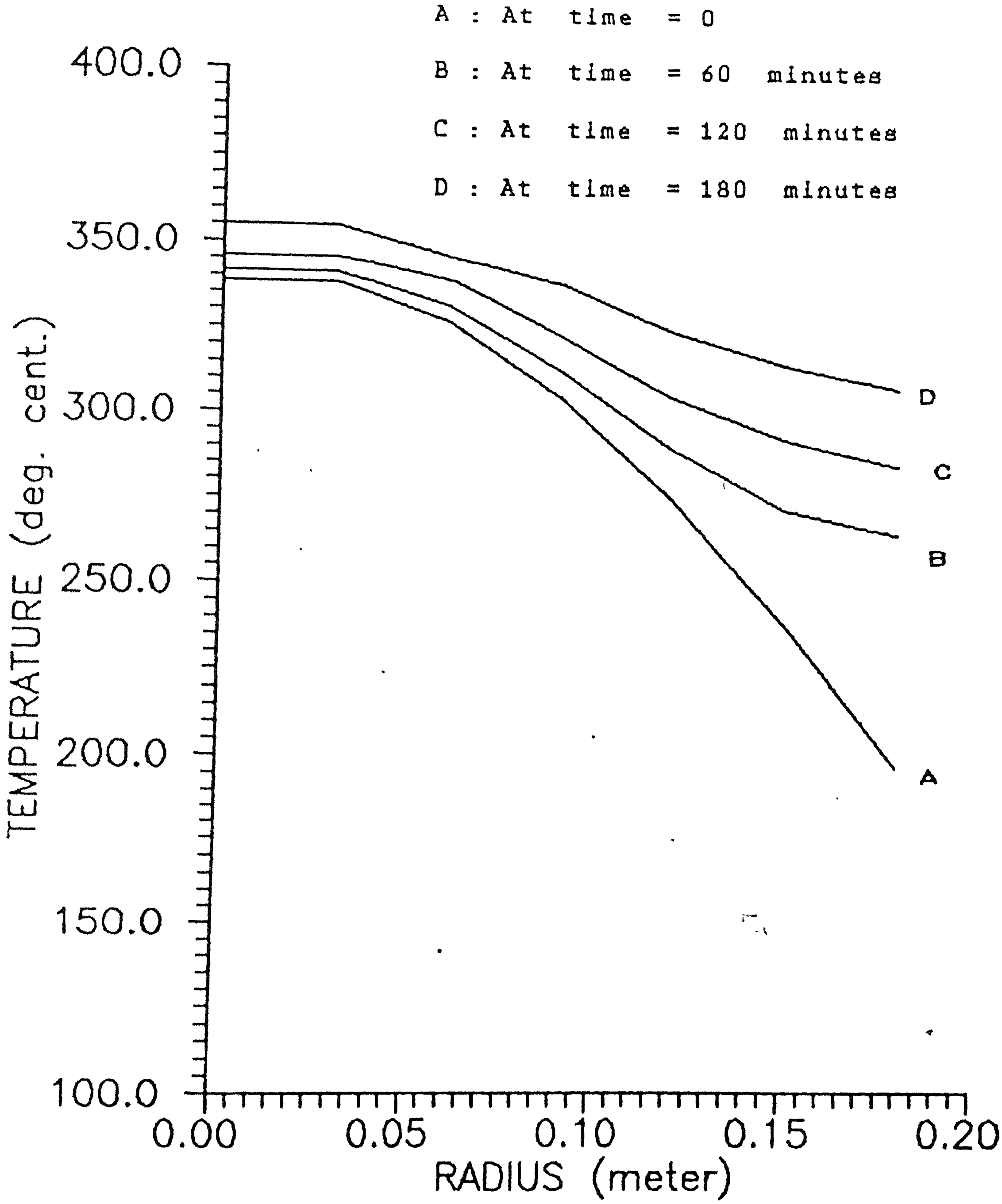

Fig. 4.2 Variation of temperature with radius of the cylinder and time for $h = 10\ W\ m^{-2}K^{-1}$

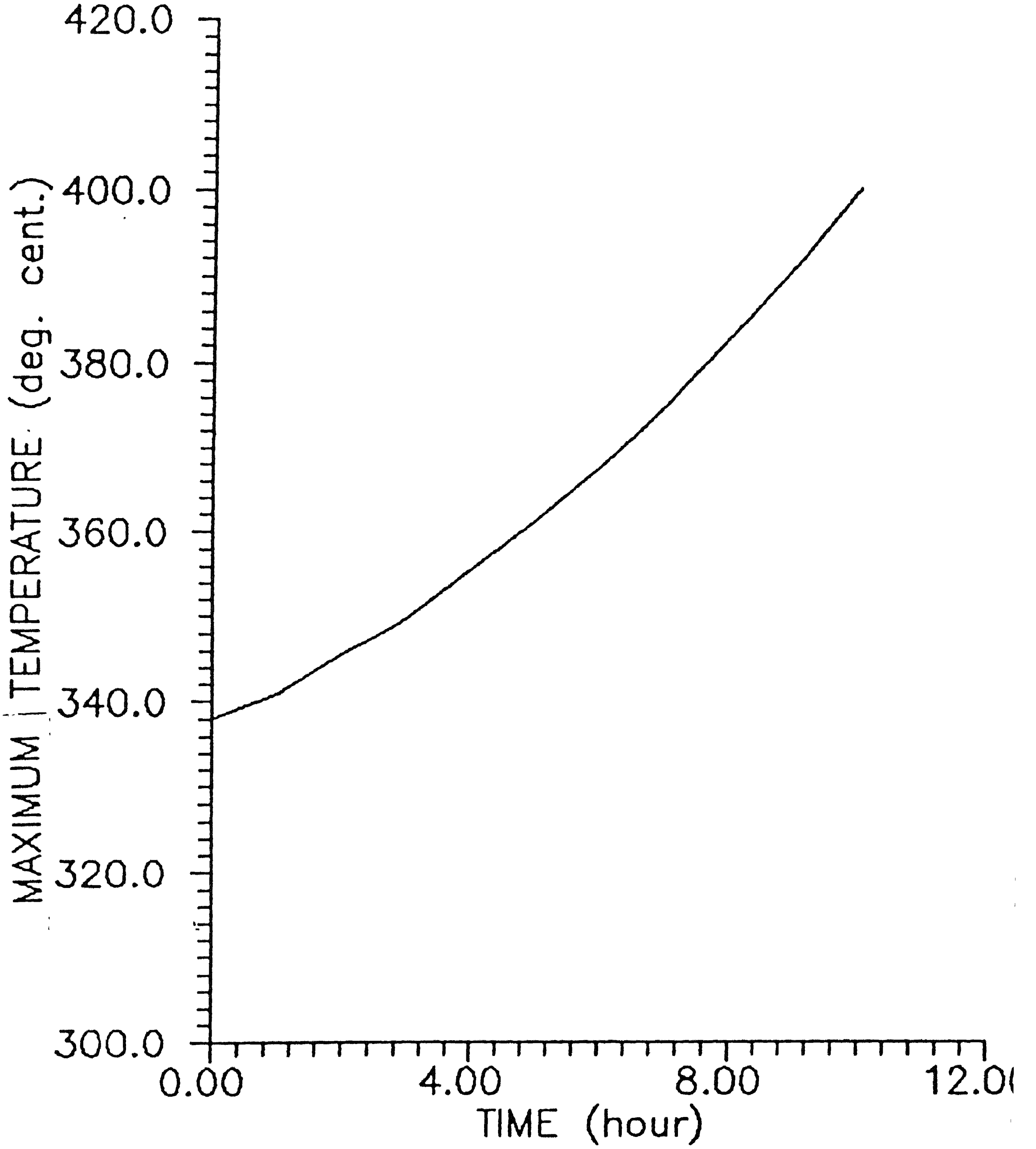

Figure - 4.3 : variation of maximum temperature with time for $h = 10\ W\ m^{-2}K^{-1}$

Chapter 5

CONCLUSIONS

1. It is necessary to store the solidified waste for an interim period of time to dissipate the decay heat to avoid melt down .

2. Constant surveillance of the waste product and its container should be done until the major portion of decay heat is dissipated .

3. Water is a better coolant than air during interim storage because of its high heat capacity ; but due to corrosion problem it is not used .

4. Actinides are relatively less important within the time frame of interim storage , because of their long half life .

5. The storage vault is to be designed on thermal as well as structural consideration .

6. The pressure loss in the cross flow of air through the storage unit is not significant . It is only about 1 percent of total head developed . However the pressure loss in the stack is around 60 % giving a stack height of 100 meter .

7. Out of three cases of heat transfer , viz. flow across the tube bundles , free convection heat transfer from a bare cylinder and free convection through an annular space , the convection heat transfer coefficient is

lowest in the third case . However the radiation heat transfer coefficient is around 160 % of convection heat transfer coefficient in case of annular space . If it is possible to have perfectly black surfaces , then the radiation heat transfer coefficient becomes 250% of convection heat transfer coefficient in this case . calculations show that the heat transfer coefficient in case of annular space can never exceed the same in case of flow across tube banks .

8. In the steady state analysis the maximum temperature always occurs at the centre of cylinder for the whole range of convection heat transfer coefficient .

9. Transient temperature distribution shows that the radial temperature distribution becomes flatter with the passage of time and the ambient temperature in the storage vault goes on rising . Calculations show that the time after which critical temperatures are expected to reach may vary from 3 to 15 hours depending upon the convection heat transfer coefficient in the building .

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